Rewarding Learning

## General Certificate of Secondary Education

Additional Mathematics

Paper 1<br>Pure Mathematics

## [G0301]



## TUESDAY 12 MAY, MORNING

## TIME

2 hours.

## INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet and the Supplementary Answer Booklet provided.
Answer all eleven questions.
At the conclusion of this examination attach the Supplementary Answer Booklet to your Answer Booklet using the treasury tag supplied.

## INFORMATION FOR CANDIDATES

The total mark for this paper is 100
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
A copy of the formulae list is provided.

## Answer all eleven questions

1 (i) Using the axes and scales in Fig. 1 in your Supplementary Answer Booklet, sketch the graph of $y=\cos x$ for $0^{\circ} \leqslant x \leqslant 360^{\circ}$.
(ii) Hence, using the axes and scales in Fig. 2 in your Supplementary Answer Booklet, sketch the graph of $y=\cos \left(x+90^{\circ}\right)$ for $0^{\circ} \leqslant x \leqslant 360^{\circ}$.

2 (i) Solve the equation

$$
\sin \theta=-0.6
$$

$$
\begin{equation*}
\text { for }-180^{\circ}<\theta \leqslant 180^{\circ} \tag{2}
\end{equation*}
$$

(ii) Hence solve the equation

$$
\begin{equation*}
\sin \left(\frac{x}{2}-20^{\circ}\right)=-0.6 \tag{2}
\end{equation*}
$$

for $-360^{\circ}<x \leqslant 360^{\circ}$.

3 (i) Find $\mathbf{A}^{-1}$ where $\mathbf{A}=\left[\begin{array}{ll}5 & -3 \\ 7 & -2\end{array}\right]$
(ii) Hence, using a matrix method, solve the following simultaneous equations for $x$ and $y$.

$$
\begin{align*}
& 5 x-3 y=6 \\
& 7 x-2 y=-7 \tag{4}
\end{align*}
$$

4 (a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$ when $y=5 x^{3}-\frac{3}{x^{5}}$
(b) Find $\int\left(9 x^{2}+\frac{2}{9 x^{2}}-4\right) \mathrm{d} x$.
$5 \quad$ Fig. 3 shows a sketch of the graph of $y=2 x^{3}-3 x^{2}+3$


Fig. 3

P and Q are two points on the curve at which the tangents are parallel to the line $y=12 x+5 . \mathrm{P}$ is to the left of Q .
(i) Find the coordinates of the points P and Q .
(ii) Find the equation of the tangent at Q .

6 (i) Show that

$$
\frac{x-4}{x-5}-\frac{2-3 x}{4 x+1}
$$

can be written as

$$
\begin{equation*}
\frac{7 x^{2}-32 x+6}{4 x^{2}-19 x-5} \tag{4}
\end{equation*}
$$

(ii) Hence, or otherwise, solve the equation

$$
\begin{equation*}
\frac{x-4}{x-5}-\frac{2-3 x}{4 x+1}=2 \tag{4}
\end{equation*}
$$

7 (a) Write $3 \log p+4 \log q$ as a single logarithm.
(b) Write $\log \frac{\sqrt{c}}{d}$ in terms of $\log c$ and $\log d$.
(c) Solve the equation

$$
6^{(2-3 x)}=15
$$

giving your answer correct to 3 decimal places.

8 A car is travelling along a straight road XY on level ground. From points A and B on the road, straight roads lead to a railway station S . The distances AB and AS are 7.25 km and 3.82 km respectively and the angle SAB is $52.75^{\circ}$, as shown in Fig. 4.


Fig. 4
(i) Find the distance SB.
(ii) Calculate the size of the angle $\mathrm{S} \hat{\mathrm{BA}}$.

A straight railway line passes through $S$ and meets the road at a level crossing $L$, where the distance BL is 8.50 km .
(iii) Find the size of the angle S $\hat{B} L$.
(iv) Find the distance SL.

At the instant the car passes B on the road a train passes through the station S . The train travels towards $L$ at a constant speed of $110 \mathrm{~km} / \mathrm{h}$. The car arrives at L just as the train is passing through the level crossing.
(v) Calculate the average speed of the car between B and L .

9 A pupil in a physics class carried out an experiment to test how the period of oscillation of an object oscillating at the end of a spring depended on the mass of the object. The experiment was carried out several times with different masses, $M$ grams, and the corresponding periods of oscillation, $T$ seconds, were recorded. The results are given in Table 1.

Table 1

| Mass | Period <br> $T(\mathrm{~s})$ |
| :---: | :---: |
| $M(\mathrm{~g})$ | 1.12 |
| 25 | 1.41 |
| 40 | 1.49 |
| 45 | 1.90 |
| 75 | 2.13 |

It is believed that a relationship of the form

$$
T=k M^{n}
$$

exists between $T$ and $M$, where $k$ and $n$ are constants.
(i) Using Fig. 5 in your Supplementary Answer Booklet, verify this relationship by drawing a suitable straight line graph, using values correct to 3 decimal places. Label the axes clearly.
(ii) Hence, or otherwise, obtain values for $k$ and $n$. Give your answers correct to 2 decimal places.
(iii) Use the formula $T=k M^{n}$ with the values you obtained for $k$ and $n$ to calculate the period of the oscillations when a mass of 80 grams is attached.
(iv) Use the formula $T=k M^{n}$ with the values you obtained for $k$ and $n$ to calculate the mass needed to be attached to the string to produce a period of 1.2 seconds.
(v) If the formula was to be used to calculate the period of the oscillations when a mass of 150 grams was attached to the string, what assumption would need to be made?

10 A photographic shop prints digital photos in small, standard and large sizes.
John ordered 40 small, 30 standard and 20 large prints. The total cost was $£ 28$.
Let $x, y$ and $z$ represent the costs, in pence, of a small, a standard and a large print respectively.
(i) Show that $x, y$ and $z$ satisfy the equation

$$
\begin{equation*}
4 x+3 y+2 z=280 \tag{1}
\end{equation*}
$$

Mary ordered 42 small, 36 standard and 6 large prints and the total cost was $£ 24$.
(ii) Show that $x, y$ and $z$ also satisfy the equation

$$
\begin{equation*}
7 x+6 y+z=400 \tag{1}
\end{equation*}
$$

If more than 50 small prints are ordered at the same time the cost of each small print is reduced by 10 pence. Similarly, if more than 50 standard prints are ordered the cost of each standard print is reduced by $\frac{1}{3}$

Nuala ordered 72 small, 96 standard and 24 large prints and the total cost was $£ 40.80$
(iii) Show that $x, y$ and $z$ also satisfy the equation

$$
\begin{equation*}
9 x+8 y+3 z=600 \tag{3}
\end{equation*}
$$

(iv) Solve these equations, showing clearly each stage of your solution.
(v) How much would John and Mary have saved if they had put in a combined order rather than two separate orders?

11 A curve is defined by the equation

$$
y=2 x^{3}+a x^{2}+b x
$$

where $a$ and $b$ are constants.
The curve has a turning point at $(-2,44)$.
(i) Show that $a=-3$ and $b=-36$
(ii) Hence find the coordinates of the other turning point.
(iii) Identify each turning point as either a maximum or a minimum point.
(iv) Find, to 1 decimal place where appropriate, the coordinates of the points where this curve crosses the $x$-axis.
(v) Sketch the curve using Fig. 6 in your Supplementary Answer Booklet.
(vi) Find the area enclosed between this curve, the $x$-axis and the line $x=1$


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## SUPPLEMENTARY <br> ANSWER BOOKLET

1 (i) Sketch the graph of $y=\cos x$ on the axes in Fig. 1 below.


Fig. 1
(ii) Sketch the graph of $y=\cos \left(x+90^{\circ}\right)$ on the axes in Fig. 2 below.


Fig. 2

9 Draw a suitable straight line graph using the axes and scales in Fig. 5 below.
Label the axes.


Fig. 5

11 Sketch the graph of $y=2 x^{3}-3 x^{2}-36 x$ in Fig. 6.


Fig. 6

