GCSE

Additional Mathematics

Summer 2009

Mark Schemes

Issued: October 2009

NORTHERN IRELAND GENERAL CERTIFICATE OF SECONDARY EDUCATION (GCSE)
AND NORTHERN IRELAND GENERAL CERTIFICATE OF EDUCATION (GCE)

MARK SCHEMES (2009)

Foreword

Introduction

Mark Schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of 16- and 18-year-old students in schools and colleges. The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes therefore are regarded as a part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

The Council hopes that the mark schemes will be viewed and used in a constructive way as a further support to the teaching and learning processes.

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Rewarding Learning

General Certificate of Secondary Education
Summer 2009

Additional Mathematics

Paper 1
Pure Mathematics

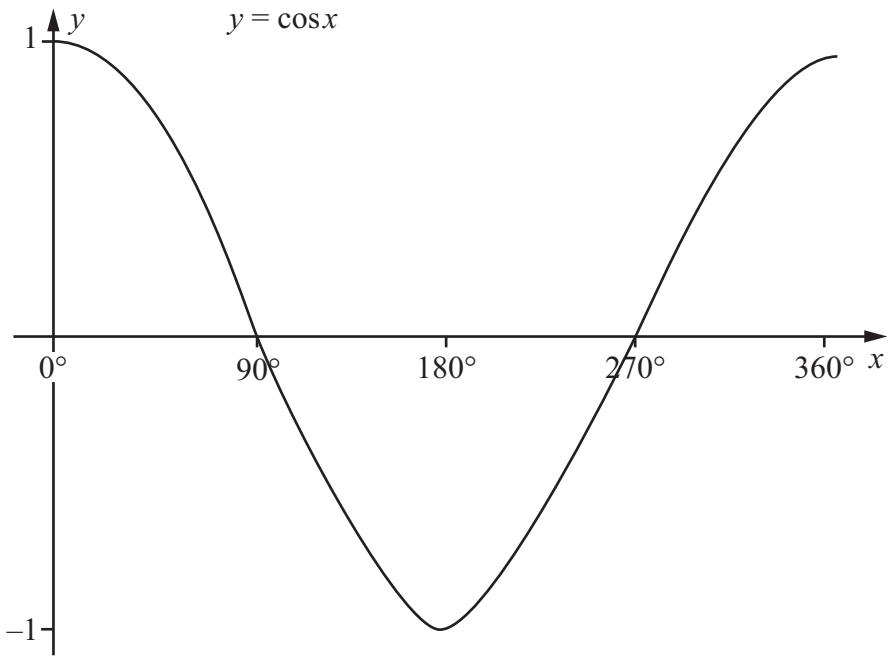
[G0301]

TUESDAY 12 MAY, MORNING

MARK SCHEME

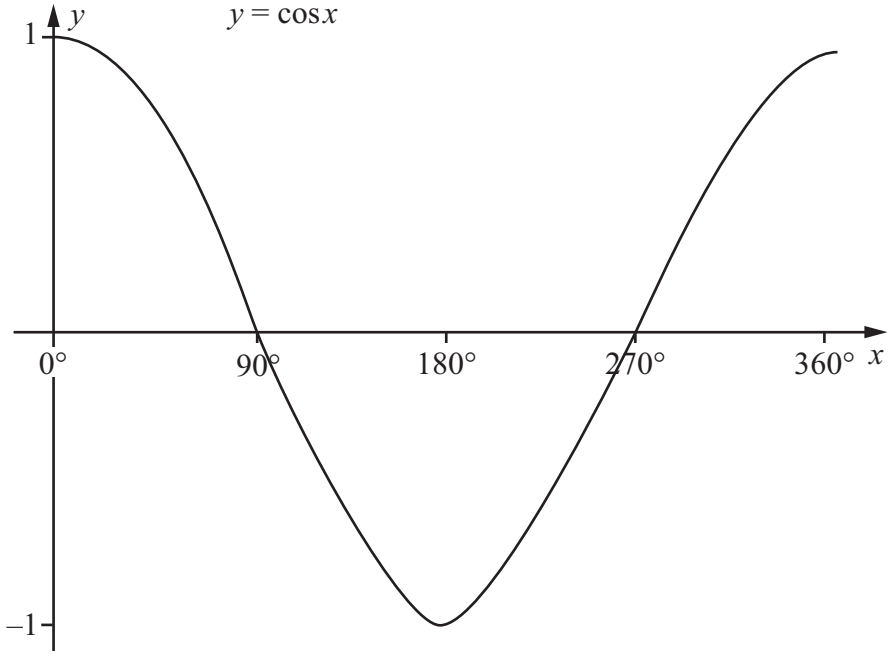
	AVAILABLE MARKS

1 **(i)**



M1, W1

(ii)



M1, W1

4

2 **(i)** $\sin\theta = -0.6$

$\therefore\ \theta = -36.87°$ **or** $-143.13°$ ($-37°$ **or** $-143°$ to nearest degree) 2 × MW1

(ii) $\sin\left(\frac{x}{2} - 20°\right) = -0.6$

From **(i)**

$\frac{x}{2} - 20° = -36.87°$ **or** $-143.13°$ M1

$\therefore\ \frac{x}{2} = -16.87°$ **or** $-123.13°$

$\therefore\ x = -33.74°$ **or** $-246.26°$ ($-34°$ **or** $-246°$ to nearest degree) W1

4

			AVAILABLE MARKS
3	**(i)** $\mathbf{A} = \begin{bmatrix} 5 & -3 \\ 7 & -2 \end{bmatrix}$		
	$\therefore \det \mathbf{A} = (5)(-2) - (7)(-3) = 11$		
	$\mathbf{A}^{-1} = \frac{1}{11}\begin{bmatrix} -2 & 3 \\ -7 & 5 \end{bmatrix}$	2 × MW1	
	(ii) $5x - 3y = 6$		
	$7x - 2y = -7$		
	$\therefore \begin{bmatrix} 5 & -3 \\ 7 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -7 \end{bmatrix}$	M1	
	$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ 7 & -2 \end{bmatrix}^{-1}\begin{bmatrix} 6 \\ -7 \end{bmatrix}$	M2	
	$= \frac{1}{11}\begin{bmatrix} -2 & 3 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 6 \\ -7 \end{bmatrix}$		
	$= \frac{1}{11}\begin{bmatrix} -33 \\ 77 \end{bmatrix} = \begin{bmatrix} -3 \\ -7 \end{bmatrix}$		
	$\therefore x = -3,\ y = -7$	W1	6
4	**(a)** $y = 5x^3 - \frac{3}{x^5}$		
	$\frac{dy}{dx} = 15x^2 + 15x^{-6}$	2 × MW1	
	or $15x^2 + \frac{15}{x^6}$		
	(b) $\int \left(9x^2 + \frac{2}{9x^2} - 4\right) dx$		
	$= 3x^3 - \frac{2}{9}x^{-1} - 4x + c$	4 × MW1	
	or $3x^3 - \frac{2}{9x} - 4x + c$		6

			AVAILABLE MARKS
5	**(i)** $y = 2x^3 - 3x^2 + 3$		
	$\frac{dy}{dx} = 6x^2 - 6x$	MW1	
	So $6x^2 - 6x = 12$	M1	
	$6x^2 - 6x - 12 = 0$		
	$x^2 - x - 2 = 0$		
	$(x - 2)(x + 1) = 0$	M1	
	$x = 2$ **or** -1		
	$y = 7$ **or** -2		
	So the points are		
	P(–1, –2), Q(2, 7)	M1, W1	
	(ii) $y = mx + c$		
	$y = 12x + c$		
	$7 = 24 + c$	MW1	
	$c = -17$		
	$y = 12x - 17$	W1	7
6	**(i)** $\frac{x-4}{x-5} - \frac{2-3x}{4x+1}$		
	$= \frac{(x-4)(4x+1) - (2-3x)(x-5)}{(x-5)(4x+1)}$	M2	
	$= \frac{(4x^2 - 15x - 4) - (-3x^2 + 17x - 10)}{4x^2 - 19x - 5}$	W1, W1	
	$= \frac{7x^2 - 32x + 6}{4x^2 - 19x - 5}$		
	(ii) $\frac{x-4}{x-5} - \frac{2-3x}{4x+1} = 2$		
	$\therefore \frac{7x^2 - 32x + 6}{4x^2 - 19x - 5} = 2$		
	$\therefore 7x^2 - 32x + 6 = 2(4x^2 - 19x - 5)$	M2	
	$= 8x^2 - 38x - 10$		
	$\therefore x^2 - 6x - 16 = 0$	W1	
	$\therefore (x + 2)(x - 8) = 0$		
	$\therefore x = -2$ **or** $x = 8$	W1	8

			AVAILABLE MARKS
7	**(a)** $3 \log p + 4 \log q$		
	$= \log p^3 + \log q^4$	MW1	
	$= \log p^3 q^4$	MW1	
	(b) $\log \frac{\sqrt{c}}{d}$		
	$= \log \sqrt{c} - \log d$	MW1	
	$= \frac{1}{2} \log c - \log d$	MW1	
	(c) $6^{2-3x} = 15$		
	$\log 6^{2-3x} = \log 15$	M1	
	$(2 - 3x) \log 6 = \log 15$	M1	
	$2 - 3x = \frac{\log 15}{\log 6}$	M1	
	$2 - 3x = 1.5113$		
	$-3x = -0.4886$		
	$x = 0.163$	W1	8
8	**(i)** $SB^2 = AS^2 + AB^2 - 2\,AS.AB \cos S\hat{A}B$		
	$= 3.82^2 + 7.25^2 - 2 \times 3.82 \times 7.25 \cos 52.75$	M2	
	$\therefore SB = 5.80\,km$	W1	
	(ii) $\frac{\sin S\hat{B}A}{AS} = \frac{\sin S\hat{A}B}{SB}$		
	$\therefore \sin S\hat{B}A = \frac{3.82 \sin 52.75}{5.80}$	M2	
	$\therefore S\hat{B}A = 31.62°$	W1	
	(iii) $S\hat{B}L = 180° - 31.62° = 148.38°$	MW1	
	(iv) $SL^2 = SB^2 + BL^2 - 2 \times SB \times BL \cos S\hat{B}L$		
	$= 5.8^2 + 8.5^2 - 2 \times 5.8 \times 8.5 \cos 148.38$	M2	
	$\therefore SL = 13.78\,km$	W1	
	(v) Time for train to reach L $= \frac{\text{distance}}{\text{speed}}$		
	$= \frac{13.78}{110} = 0.125\ h$	MW1	
	$\therefore$ average speed of car $= \frac{\text{distance}}{\text{time}}$		
	$= \frac{8.5}{0.125} = 68\ km/h$	MW1	12

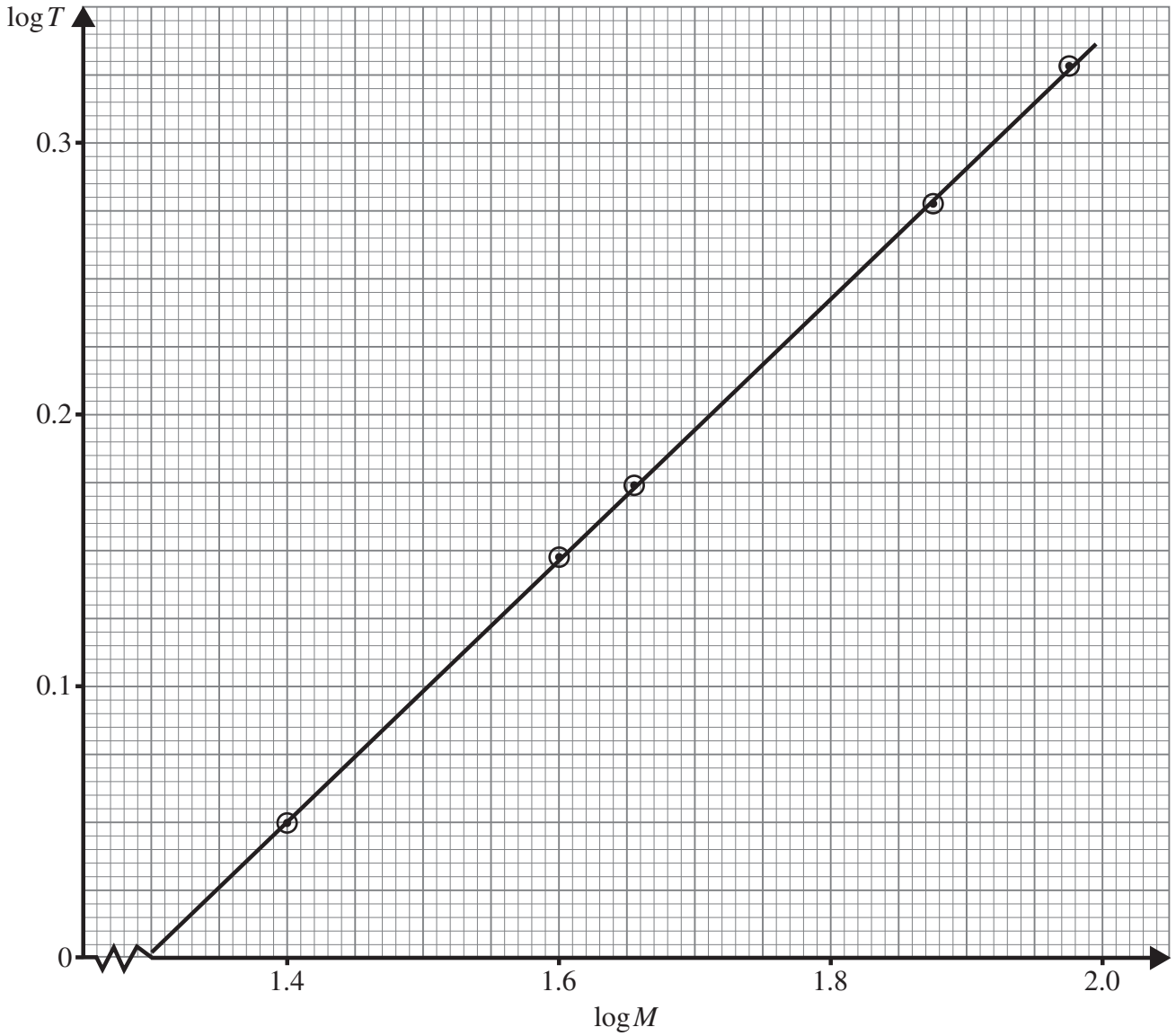
AVAILABLE MARKS

9 (i) $T = kM^n$

$\log T = n \log M + \log k$ M1

$\log M$	$\log T$
1.398	0.049
1.602	0.149
1.653	0.173
1.875	0.279
1.978	0.328

M1, W1



AVAILABLE MARKS

W1 labels
W1 points
W1 straight line

			AVAILABLE MARKS
(ii)	$n = \frac{0.328 - 0.049}{1.978 - 1.398} = \frac{0.279}{0.58}$	M1	
	$= 0.48$	W1	
	$T = kM^{0.48}$		
	$1.12 = k(25)^{0.48}$	M1	
	$1.12 = 4.69k$		
	$k = 0.24$	W1	
(iii)	$T = 0.24M^{0.48}$		
	$T = 0.24 \times 80^{0.48}$		
	$T = 1.97$ sec	MW1	
(iv)	$1.2 = 0.24M^{0.48}$		
	$M^{0.48} = 5$	M1	
	$M = \sqrt[0.48]{5}$ **or** $0.48 \log M = \log 5$		
	$M = 28.6$	W1	
(v)	Assume that the formula holds outside the range of the given values.	W1	14

	AVAILABLE MARKS

10 (i) $40x + 30y + 20z = 2800$ MW1

$\therefore\ 4x + 3y + 2z = 280$

(ii) $42x + 36y + 6z = 2400$ MW1

$\therefore\ 7x + 6y + z = 400$

(iii) $72(x - 10) + 96\left(\frac{2}{3}y\right) + 24z = 4080$ M1, M1

$\therefore\ 72x - 720 + 64y + 24z = 4080$

$\therefore\ 72x + 64y + 24z = 4800$ W1

$\therefore\ 9x + 8y + 3z = 600$

(iv) $4x + 3y + 2z = 280$ (1)

$7x + 6y + z = 400$ (2)

$9x + 8y + 3z = 600$ (3)

$2 \times (2) - (1) \rightarrow 10x + 9y = 520$ (4)

$3 \times (2) - (3) \rightarrow 12x + 10y = 600$ M2, W2

$\therefore\ 6x + 5y = 300$ (5)

$9 \times (5) - 5 \times (4) \rightarrow 4x = 100$ M2

$\therefore\ x = 25$

$\therefore\ y = \dfrac{300 - 6x}{5} = 30$

$\therefore\ z = 400 - 7x - 6y = 45$ M1, W1

So costs are: small – 25p
standard – 30p
large – 45p

(v) John and Mary would save on small and standard prints as together they have 82 small and 66 standard prints.

Amount saved on each small print = 10p

Amount saved on each standard print = $\frac{1}{3}(30) = 10$p M1

So total amount saved = $82 \times 10\text{p} + 66 \times 10\text{p}$

= £14.80 W1 15

AVAILABLE MARKS

11 **(i)** $y = 2x^3 + ax^2 + bx$

$44 = -16 + 4a - 2b$ M1

$4a - 2b = 60$ ①

$\frac{dy}{dx} = 6x^2 + 2ax + b$ MW1

$24 - 4a + b = 0$ M1

$-4a + b = -24$ ②

① + ② $\rightarrow -b = 36$

$\therefore b = -36$

$4a + 72 = 60$

$\therefore 4a = -12, a = -3$ M1

(ii) $y = 2x^3 - 3x^2 - 36x$

$\frac{dy}{dx} = 6x^2 - 6x - 36 = 0$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$ M1

$x = 3$ $y = -81$

$(3, -81)$ W1

(iii) $\frac{d^2y}{dx^2} = 12x - 6$

$x = 3 : \frac{d^2y}{dx^2} = 36 - 6 = 30$ $(3, -81)$ Minimum MW1

$x = -2 : \frac{d^2y}{dx^2} = -24 - 6 = -30$ $(-2, 44)$ Maximum MW1

(iv) $2x^3 - 3x^2 - 36x = 0$ M1

$x(2x^2 - 3x - 36) = 0$

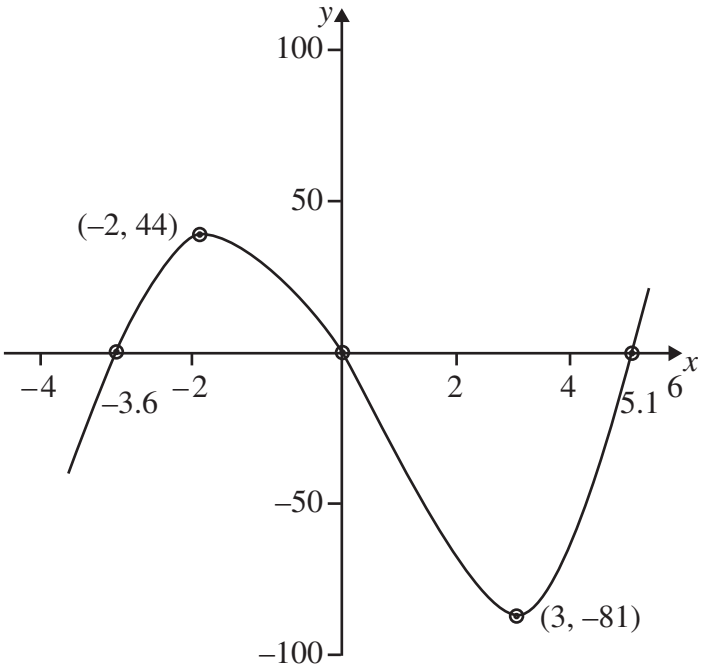
$2x^2 - 3x - 36 = 0$

$x = \frac{3 \pm \sqrt{9 + 288}}{4}$ M1

$x = \frac{3 \pm 17.23}{4}$

$x = 5.1$ **or** -3.6

$(0, 0)$ $(5.1, 0)$ $(-3.6, 0)$ W1

		AVAILABLE MARKS
(v) 	W1 max/min W1 axes	
(vi) $\int_0^1 (2x^3 - 3x^2 - 36x)\,dx$	M1	
$= \left[\frac{1}{2}x^4 - x^3 - 18x^2\right]_0^1$	MW1	
$= (0.5 - 1 - 18) - (0) = -18.5$		
Area = 18.5	W1	16
	Total	**100**

Rewarding Learning

General Certificate of Secondary Education
Summer 2009

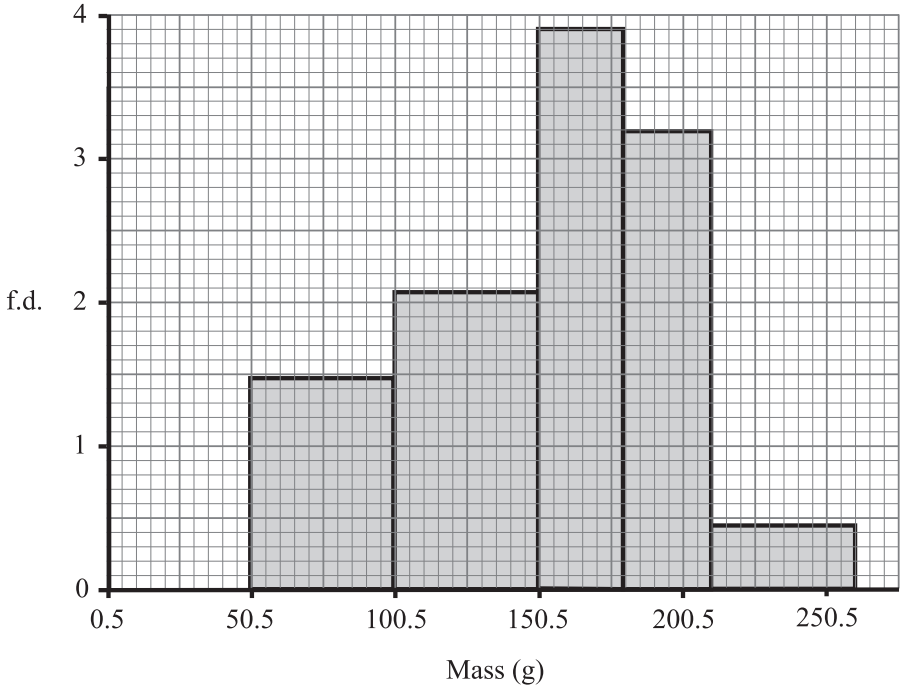
Additional Mathematics

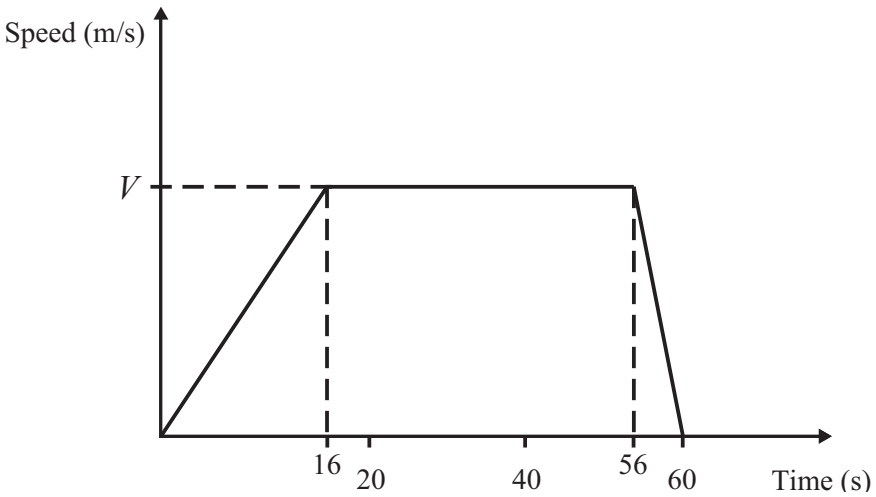
Paper 2
Mechanics and Statistics

[G0302]

FRIDAY 15 MAY, MORNING

MARK SCHEME

		AVAILABLE MARKS
1	**(i)** Using $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{at}$	
	$(5\mathbf{i} - \mathbf{j}) = (2\mathbf{i} + 3\mathbf{j}) + \boldsymbol{a}\,(2)$ MW1	
	$2\boldsymbol{a} = 3\mathbf{i} - 4\mathbf{j}$	
	$\boldsymbol{a} = \left(\frac{3}{2}\mathbf{i} - 2\mathbf{j}\right)$ m/s^2 W1	
	(ii) Using $\boldsymbol{F} = \mathrm{m}\boldsymbol{a}$	
	$\boldsymbol{F} = 5\left(\frac{3}{2}\mathbf{i} - 2\mathbf{j}\right)$ MW1	
	$\boldsymbol{F} = \left(\frac{15}{2}\mathbf{i} - 10\mathbf{j}\right)$ N	
	(iii) $\lvert\boldsymbol{F}\rvert = \sqrt{\left(\frac{15}{2}\right)^2 + (-10)^2} = \sqrt{\frac{225}{4} + 100} = \sqrt{\frac{625}{4}}$ MW1	4
	$\lvert\boldsymbol{F}\rvert = \frac{25}{2} = 12.5$N	
2	class width 50 50 30 30 50 M1W1	
	f.d. 1.48 2.08 3.9 3.2 0.44	
		
	W1 axes W1 cts scale at boundaries W1 heights	
		5

				AVAILABLE MARKS
3	**(i)**		W1 W1	
	(ii)	Distance travelled $= \frac{1}{2}(16)(V) + 40V + \frac{1}{2}(4)(V)$		
		$= 8V + 40V + 2V$	MW2	
		$= 50V$		
		$\Rightarrow 50V = 900$	W1	
		$\Rightarrow V = 18$ m/s	W1	
	or	Distance travelled $= \frac{1}{2}(60 + 40)\ V = 50V$	MW2	
		$\Rightarrow 50V = 900$	W1	
		$\Rightarrow V = 18$m/s	W1	6
4	**(i)**	LCB = 14.5 kg, UCL = 18 kg CW = 18.5 – 14.5 = 4 kg	M1W1 for limits	
	(ii)	LCB = 5.45 cm, UCL = 8.5 cm CW = 8.55 – 5.45 = 3.1 cm	M1W1 for boundaries	
	(iii)	LCB = –4.5 °C, UCL = 2 °C CW = 2.5 + 4.5 = 7 °C	M1W1 for class widths	6

AVAILABLE MARKS

5 **(i)**

Number of books	f	mid value x	fx
11–25	14	18	252
26–35	18	30.5	549
36–45	21	40.5	850.5
46–55	5	50.5	252.5
56–65	2	60.5	121

$\text{mean} = \dfrac{\Sigma fx}{\Sigma f} = \dfrac{2025}{60} = 33.75$ M1, W1

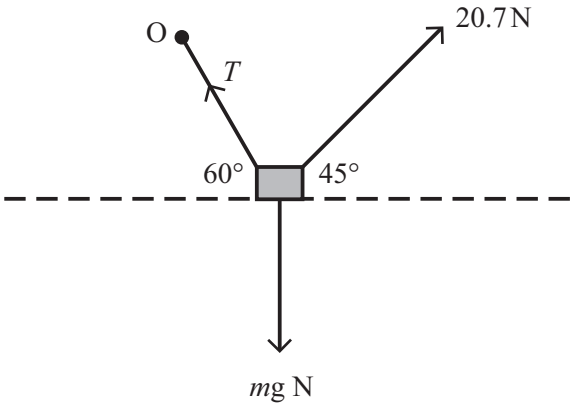
(ii) Total = 60, median class = 26–35

$\text{Median} = 25.5 + \dfrac{30 - 14}{18} \times 10$ or $25.5 + \dfrac{30.5 - 14}{18} \times 10$

(25.5) MW1
(30 or 30.5) MW1
$(\dfrac{\text{diff}}{\text{class freq}} \times \text{class width})$ MW1

= 34.4 (to 1 d.p.) = 34.7 (to 1 d.p.) MW1 — 6

6



(i) Resolve horizontally

$T\cos 60° = 20.7\cos 45°$ M1W1

$T = \dfrac{20.7\cos 45°}{\cos 60°} = 29.3\text{ N}$ (to 1 d.p.) W1

(ii) Resolve vertically

$T\sin 60° + 20.7\sin 45° = mg$

$29.3\sin 60° + 20.7\sin 45° = 10m$ M1W1

$25.4 + 14.6 = 10m$ MW1

$\Rightarrow 10m = 40.0$

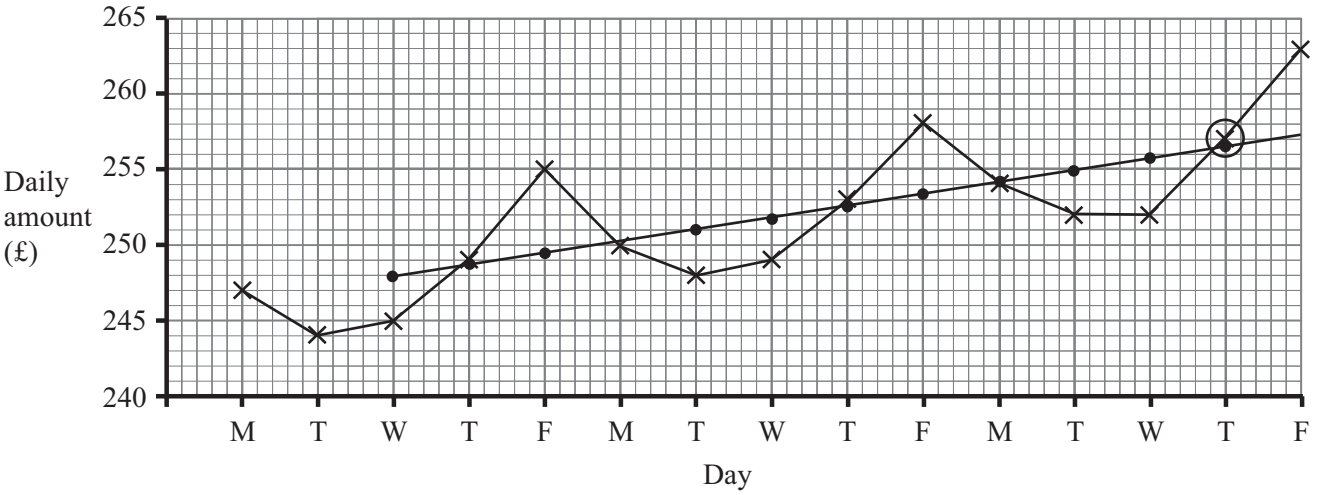
$\Rightarrow \quad m = 4\text{ kg}$ W1 — 7

AVAILABLE MARKS

7 **(i)** 5 pt moving averages

248
248.6
249.4
250.2
251
251.6
252.4
253.2
253.8
254.6
255.6 M1W1

(ii) M2 set
W1 line



AVAILABLE MARKS

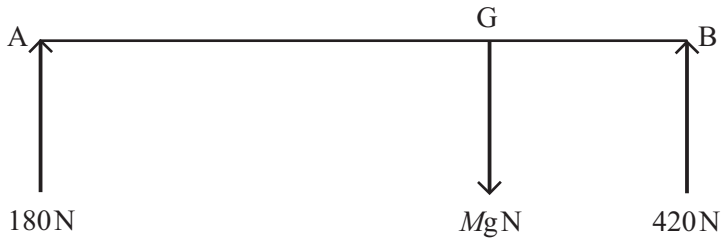
(iii) $\frac{x + 263 + 257 + 252 + 252}{5} = 256.4$ M1W1 reading

$x = £258$ M1W1

9

	AVAILABLE MARKS

8



(i) Let M kg equal the mass of the plank.

$M\text{g} = 180 + 420$ MW1

$\Rightarrow 10M = 600$

$\Rightarrow \quad M = 60\,\text{kg}$ W1

(ii) Take moments at G

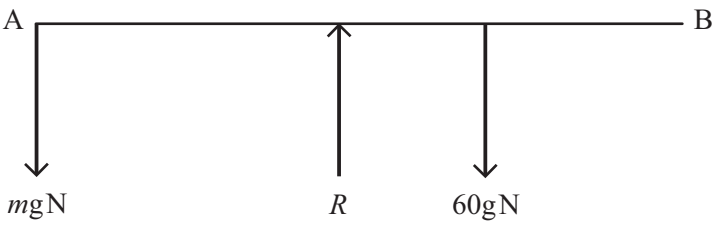
$180x = 420(8 - x)$ MW1 MW1

$\Rightarrow 18x = 336 - 42x$

$\Rightarrow 60x = 336$

$\Rightarrow \quad x = 5.6\,\text{m}$ W1

(iii)



W1 W1 W1

(iv) Take moments at the mid-point of the plank.

$m\cancel{g}(4) = 60\cancel{g}(5.6 - 4)$ MW1

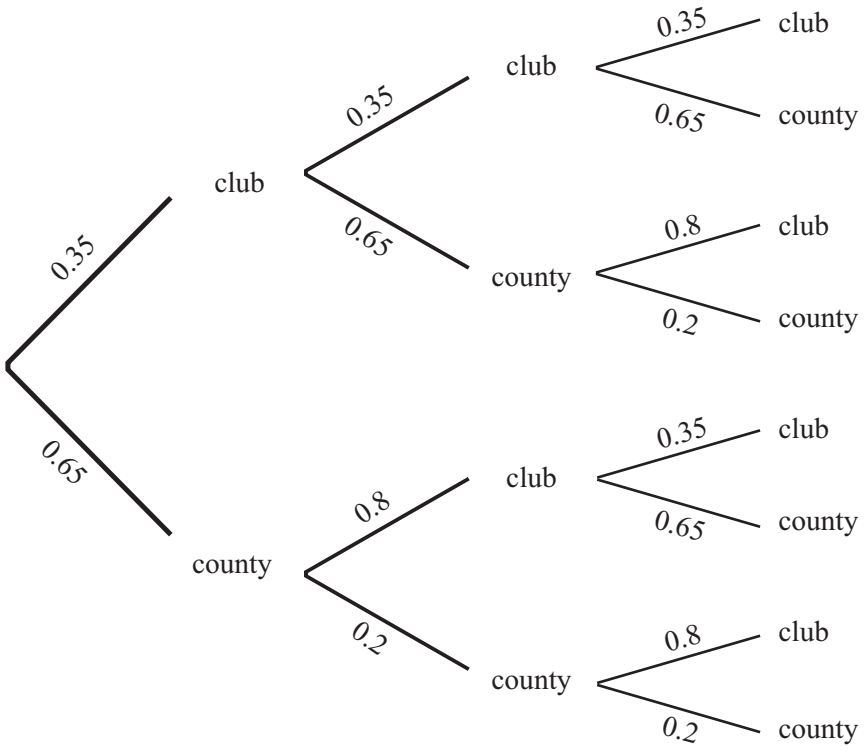
$\Rightarrow 4m = 60(1.6)$

$\Rightarrow 4m = 96$

$\Rightarrow \quad m = 24\,\text{kg}$ W1 10

	AVAILABLE MARKS

9 **(i)**

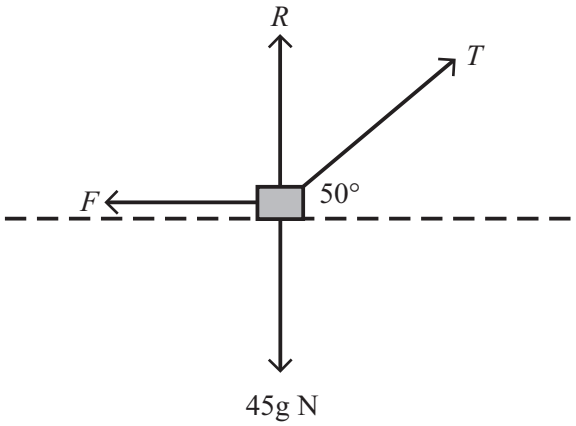


M1W2

(ii) $0.65 \times 0.2 \times 0.2 = 0.026$ M1W1

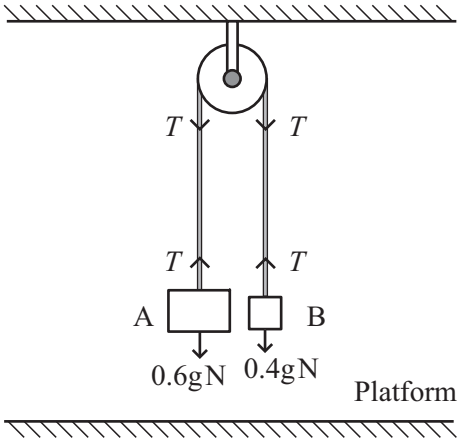
(iii) $0.65 \times 0.8 \times 0.35 + 0.35 \times 0.65 \times 0.8 + 0.35 \times 0.35 \times 0.65 = 0.444$ M2W1

(iv) $1 - P(\text{no county}) = 1 - 0.35^3 = 0.957$ M2W1 11

		AVAILABLE MARKS
10 (i)		
	W1	
	W1	
(ii)	Resolve vertically	
	$R + 175 \sin 50° = 45g$ MW1	
	$\Rightarrow R = 450 - 175 \sin 50°$	
	$\Rightarrow R = 315.94\,\text{N}$ W1	
(iii)	Resolve horizontally	
	$F = T \cos 50°$	
	$\Rightarrow F = 175 \cos 50°$ MW1	
	$\Rightarrow F = 112.49\,\text{N}$	
	But $F = \mu R$	
	$\Rightarrow 112.49 = 315.94\,\mu$ MW1	
	$\Rightarrow \mu = 0.356$ W1	
(iv)	Let R_1 = new reaction and F_1 = new frictional force	
	Resolve vertically	
	$R_1 + T \sin 35° = 45g$	
	$\Rightarrow R_1 = 450 - T \sin 35°$ MW1	
	$\Rightarrow R_1 = 349.62\,\text{N}$	
	$F_1 = \mu R_1 = 0.356 \times 349.62$ MW1	
	$\Rightarrow F_1 = 124.46\,\text{N}$	
	Accel. force $= T \cos 35° - F_1 = 18.89\,\text{N}$ MW1	
	Accel. force = mass × acceleration	
	$\Rightarrow 18.89 = 45a$	
	acceleration $= 0.42\,\text{m/s}^2$ W1	11

	AVAILABLE MARKS

11



(i) $0.6g - T = 0.6a$ MW1

$T - 0.4g = 0.4a$ MW1

$\Rightarrow 0.2g = 1.0a$

$= a = 2\,\text{m/s}^2$ W1

(ii) $T - 0.4g = 0.4a$

$\Rightarrow T = (0.4)(2) + 4$ MW1

$\Rightarrow T = 4.8\,\text{N}$

(iii) Force exerted by the string on the pulley $= 2T$

$\Rightarrow$ Force $= 2(4.8) = 9.6\,\text{N}$ MW1

(iv) Using $v = u + at$ with $u = 0$ M1W1

$v = 0 + 2(1.5) = 3\,\text{m/s}$

(v) Using $v^2 = u^2 + 2as$ with $v = 0\,, a = -10\,\text{m/s}^2$

$\Rightarrow 0 = 3^2 + 2(-10)\,s$ M1W1

$\Rightarrow 20s = 9$

$\Rightarrow s = 0.45\,\text{m}$ W1

(vi) Using $v = u + at$ with $v = 0$, $u = 3$, $a = -10$

$\Rightarrow 0 = 3 + (-10)\,t$ MW1

$\Rightarrow 10t = 3 \Rightarrow t = 0.3$ secs

Time for string to become taut again $= 2(0.3) = 0.6$ secs MW1 12

AVAILABLE MARKS

12 **(i)**

	1	7.5	3	2	10	6	7.5	5	4	9
	10	1	6.5	9	4	2	8	5	6.5	3
d^2	81	42.25	12.25	49	36	16	0.25	0	6.25	36

Ranks W2

(ii) $r = 1 - \dfrac{6(279)}{10(99)}$ Σd^2 M1W1

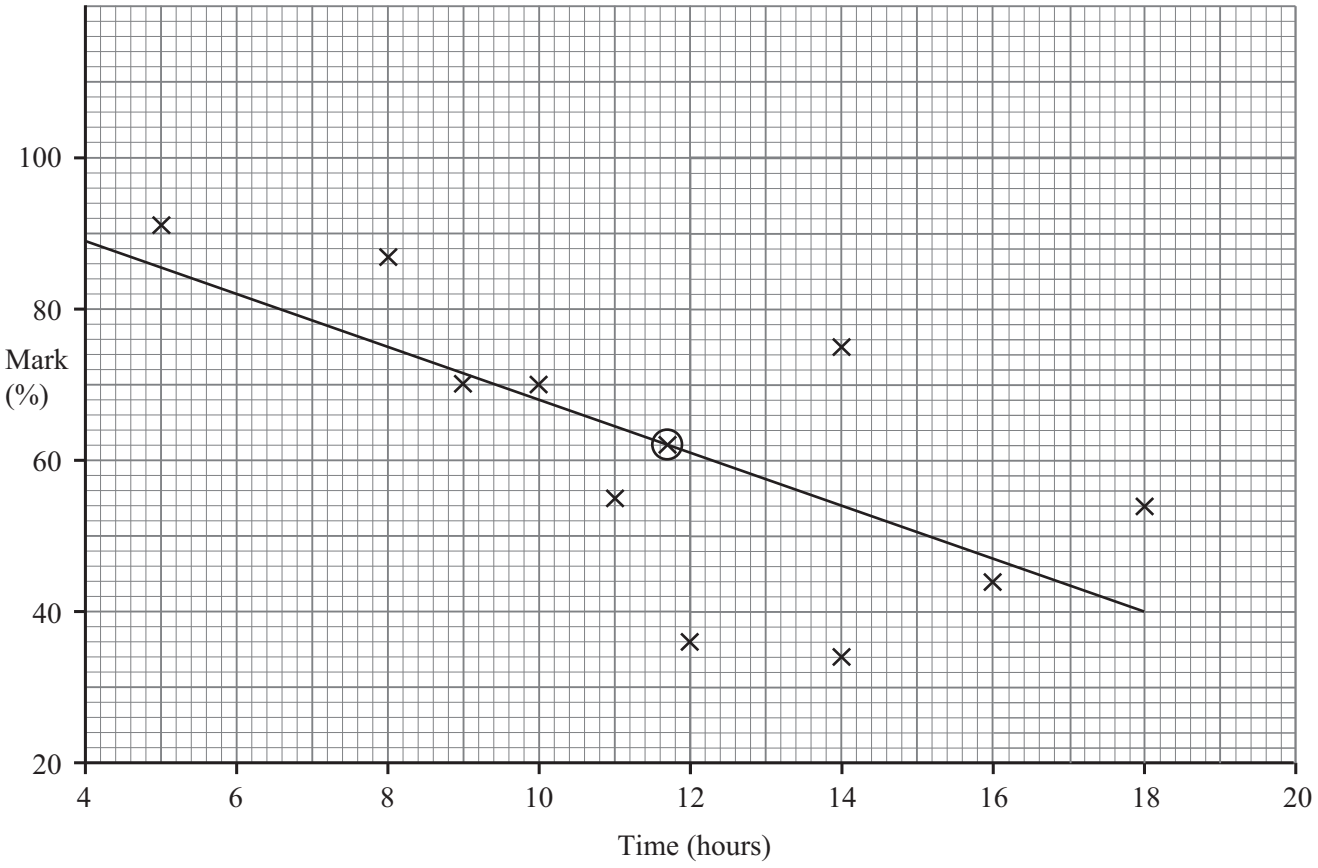
$= -0.691$ M1W1

(iii) negative correlation M1

(iv) mean hours = 11.7 mean mark = 61.6 MW1

MW1 thru means

(v) MW1 slope



AVAILABLE MARKS

(vi) $\dfrac{40 - 89}{18 - 4} = -3.5$ correct method for gradient MW1

$89 = -3.5(4) + c \ \therefore c = 103$ correct method for intercept M1

$y = -3.5x + 103$ equation MW1

both methods correct 13

Total **100**