A

AQA

Surname ____________________

Other Names ____________________

Centre Number ____________________

Candidate Number ____________________

Candidate Signature ____________________
I declare this is my own work.

A-level
MATHEMATICS

Paper 1

7357/1

Wednesday 3 June 2020 Afternoon

Time allowed: 2 hours

At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.

[Turn over]

J U N 2 0 7 3 5 7 1 0 1

- **You must have the AQA Formulae for A-level Mathematics booklet.**
- **You should have a graphical or scientific calculator that meets the requirements of the specification.**

INSTRUCTIONS

- **Use black ink or black ball-point pen. Pencil should only be used for drawing.**
- **Answer ALL questions.**
- **You must answer each question in the space provided for that question.**
- **Do NOT write on blank pages.**
- **Show all necessary working; otherwise marks for method may be lost.**
- **Do all rough work in this book. Cross through any work that you do not want to be marked.**

INFORMATION

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

ADVICE

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

DO NOT TURN OVER UNTIL TOLD TO DO SO

Answer ALL questions in the spaces provided.

1 **The first three terms, in ascending powers of x, of the binomial expansion of $(9 + 2x)^{\frac{1}{2}}$ are given by**

$$(9 + 2x)^{\frac{1}{2}} \approx a + \frac{x}{3} - \frac{x^2}{54}$$

where a is a constant.

1 (a) **State the range of values of x for which this expansion is valid.**

Circle your answer. [1 mark]

$|x| < \frac{2}{9}$

$|x| < \frac{2}{3}$

$|x| < 1$

$|x| < \frac{9}{2}$

[Turn over]

1 (b) **Find the value of a.**

Circle your answer. [1 mark]

1 2 3 9

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[Turn over]

2 A student is searching for a solution to the equation $f(x) = 0$

He correctly evaluates

$f(-1) = -1$ and $f(1) = 1$

and concludes that there must be a root between -1 and 1 due to the change of sign.

Select the function $f(x)$ for which the conclusion is INCORRECT.

Circle your answer. [1 mark]

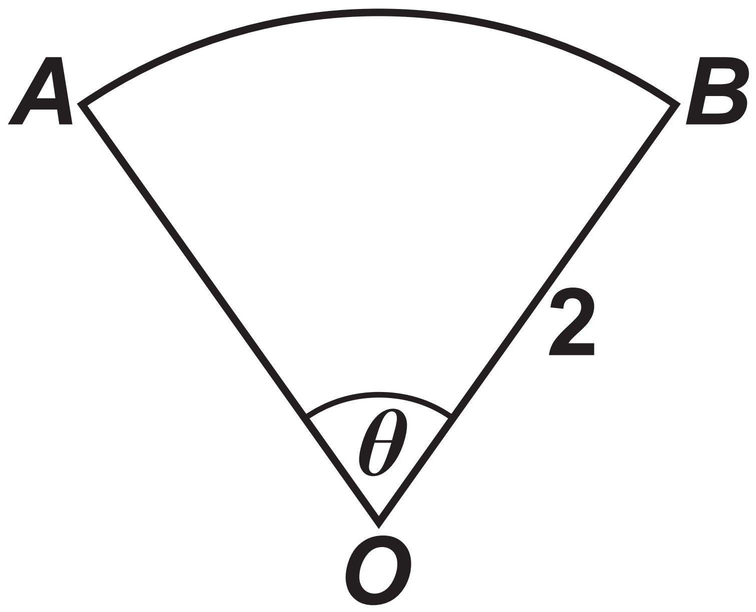
$f(x) = \frac{1}{x}$

$f(x) = x$

$f(x) = x^3$

$f(x) = \frac{2x + 1}{x + 2}$

3 The diagram shows a sector *OAB* of a circle with centre *O* and radius 2



The angle *AOB* is θ radians and the perimeter of the sector is 6

Find the value of θ

Circle your answer. [1 mark]

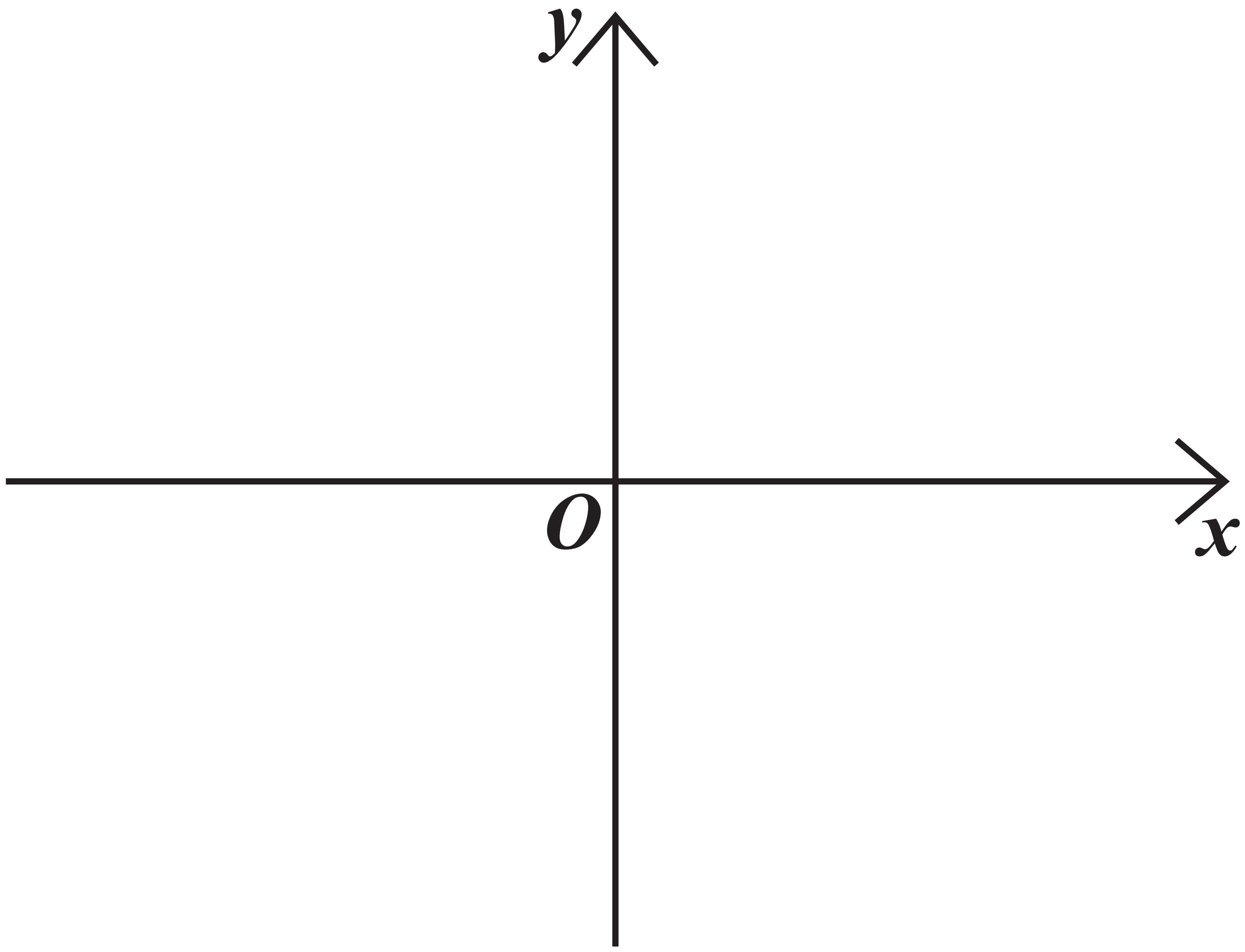
1 $\sqrt{3}$ 2 3

[Turn over]

4 (a) Sketch the graph of

$$y = 4 - |2x - 6|$$

[3 marks]



4 (b) Solve the inequality

$$4 - |2x - 6| > 2$$

[2 marks]

5 **Prove that, for integer values of n such that $0 \leq n < 4$**

$$2^{n+2} > 3^n$$

[2 marks]

[Turn over]

6 Four students, Tom, Josh, Floella and Georgia are attempting to complete the indefinite integral

$$\int \frac{1}{x}\,\mathrm{d}x \qquad \text{for } x > 0$$

Each of the students' solutions is shown below:

Tom $\int \frac{1}{x}\,\mathrm{d}x = \ln x$

Josh $\int \frac{1}{x}\,\mathrm{d}x = k\ln x$

Floella $\int \frac{1}{x}\,\mathrm{d}x = \ln Ax$

Georgia $\int \frac{1}{x}\,\mathrm{d}x = \ln x + c$

6 (a) (i) Explain what is wrong with Tom's answer. [1 mark]

6 (a) (ii) Explain what is wrong with Josh's answer. [1 mark]

6 (b) **Explain why Floella and Georgia's answers are equivalent.**
[2 marks]

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[Turn over]

7 Consecutive terms of a sequence are related by

$$u_{n+1} = 3 - (u_n)^2$$

7 (a) In the case that $u_1 = 2$

7 (a) (i) Find u_3 [2 marks]

7 (a) (ii) Find u_{50} [1 mark]

7 (b) State a different value for u_1 which gives the same value for u_{50} as found in part (a)(ii).
[1 mark]

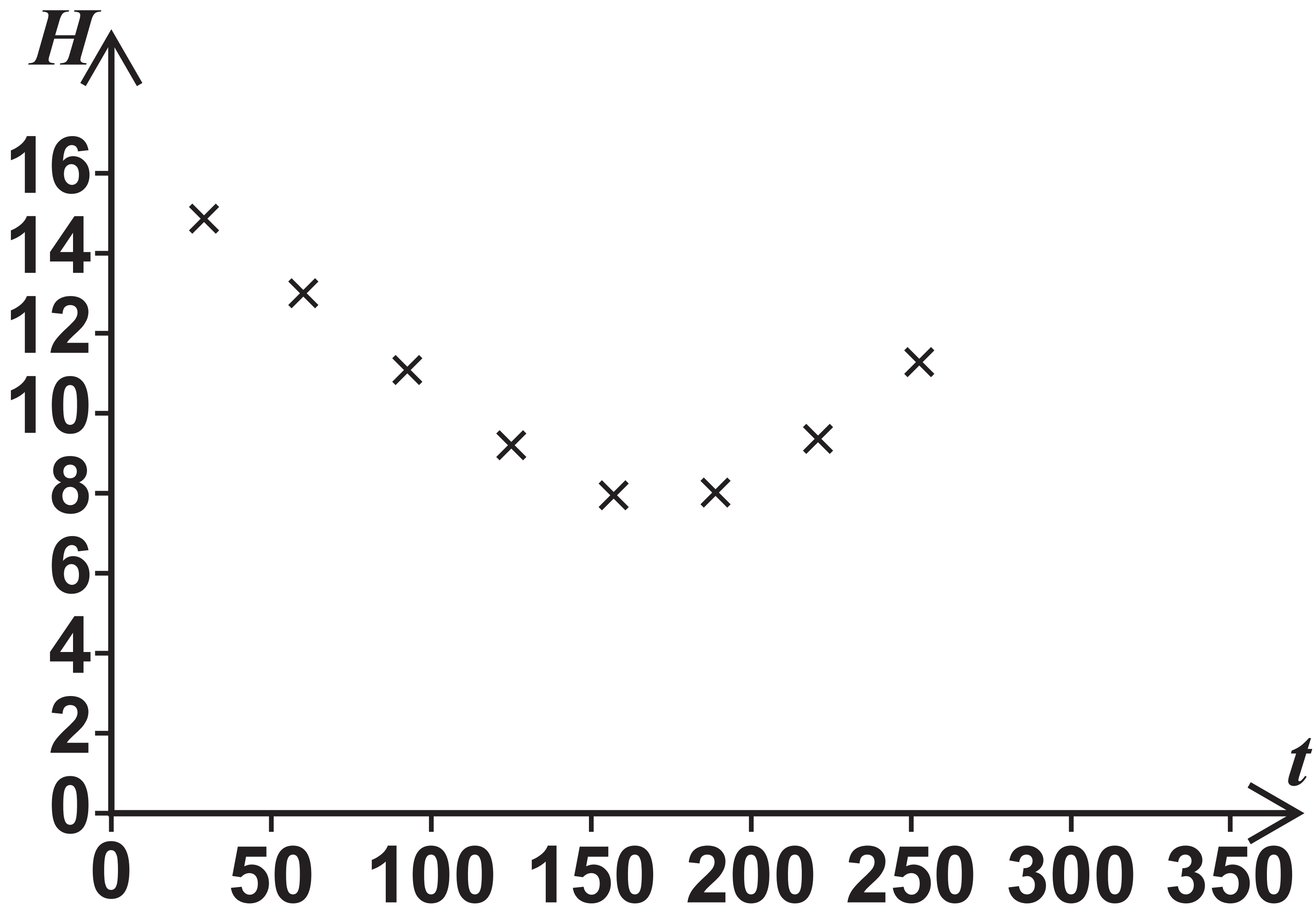
[Turn over]

8 **Mike, an amateur astronomer who lives in the South of England, wants to know how the number of hours of darkness changes through the year.**

On various days between February and September he records the length of time, H hours, of darkness along with t, the number of days after 1 January.

His results are shown in FIGURE 1 opposite.

FIGURE 1



Mike models this data using the equation

$$H = 3.87\sin\left(\frac{2\pi(t + 101.75)}{365}\right) + 11.7$$

[Turn over]

8 (a) **Find the minimum number of hours of darkness predicted by Mike's model. Give your answer to the nearest minute. [2 marks]**

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[Turn over]

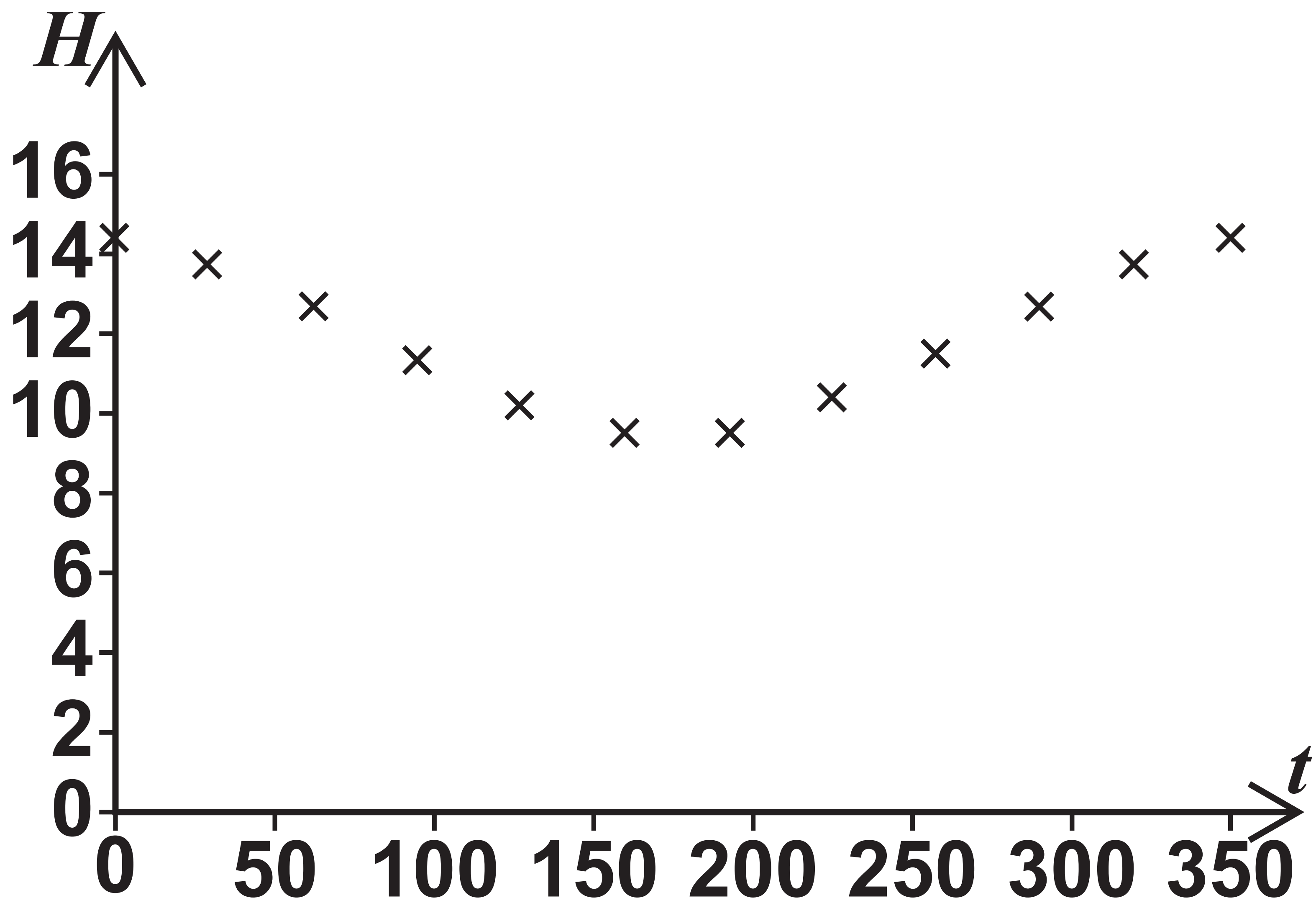
8 (b) Find the maximum number of consecutive days where the number of hours of darkness predicted by Mike's model exceeds 14 [3 marks]

[Turn over]

8 (c) Mike's friend Sofia, who lives in Spain, also records the number of hours of darkness on various days throughout the year.

Her results are shown in FIGURE 2 below.

FIGURE 2



Sofia attempts to model her data by refining Mike's model.

She decides to increase the 3.87 value, leaving everything else unchanged.

Explain whether Sofia's refinement is appropriate. [2 marks]

[Turn over]

9 Chloe is attempting to write $\dfrac{2x^2 + x}{(x+1)(x+2)^2}$ as partial fractions, with constant numerators.

Her incorrect attempt is shown below.

Step 1

$$\frac{2x^2 + x}{(x+1)(x+2)^2} \equiv \frac{A}{x+1} + \frac{B}{(x+2)^2}$$

Step 2

$$2x^2 + x \equiv A(x+2)^2 + B(x+1)$$

Step 3 Let $x = -1 \Rightarrow A = 1$
Let $x = -2 \Rightarrow B = -6$

Answer

$$\frac{2x^2 + x}{(x+1)(x+2)^2} \equiv \frac{1}{x+1} - \frac{6}{(x+2)^2}$$

9 (a) (i) By using a counter example, show that the answer obtained by Chloe cannot be correct.
[2 marks]

[Turn over]

9 (a) (ii) Explain her mistake in Step 1. [1 mark]

9 (b) Write $\dfrac{2x^2 + x}{(x + 1)(x + 2)^2}$ as partial fractions, with constant numerators. [4 marks]

[Turn over]

10 (a) **An arithmetic series is given by**

$$\sum_{r=5}^{20} (4r + 1)$$

10 (a) (i) **Write down the first term of the series. [1 mark]**

10 (a) (ii) **Write down the common difference of the series. [1 mark]**

10 (a) (iii) Find the number of terms of the series. [1 mark]

[Turn over]

10 (b) A DIFFERENT arithmetic series is given by

$$\sum_{r=10}^{100} (br + c)$$

where b and c are constants.

The sum of this series is 7735

10 (b) (i) Show that $55b + c = 85$
[4 marks]

[Turn over]

10 (b) (ii) The 40th term of the series is 4 times the 2nd term.

Find the values of b and c.
[4 marks]

[Turn over]

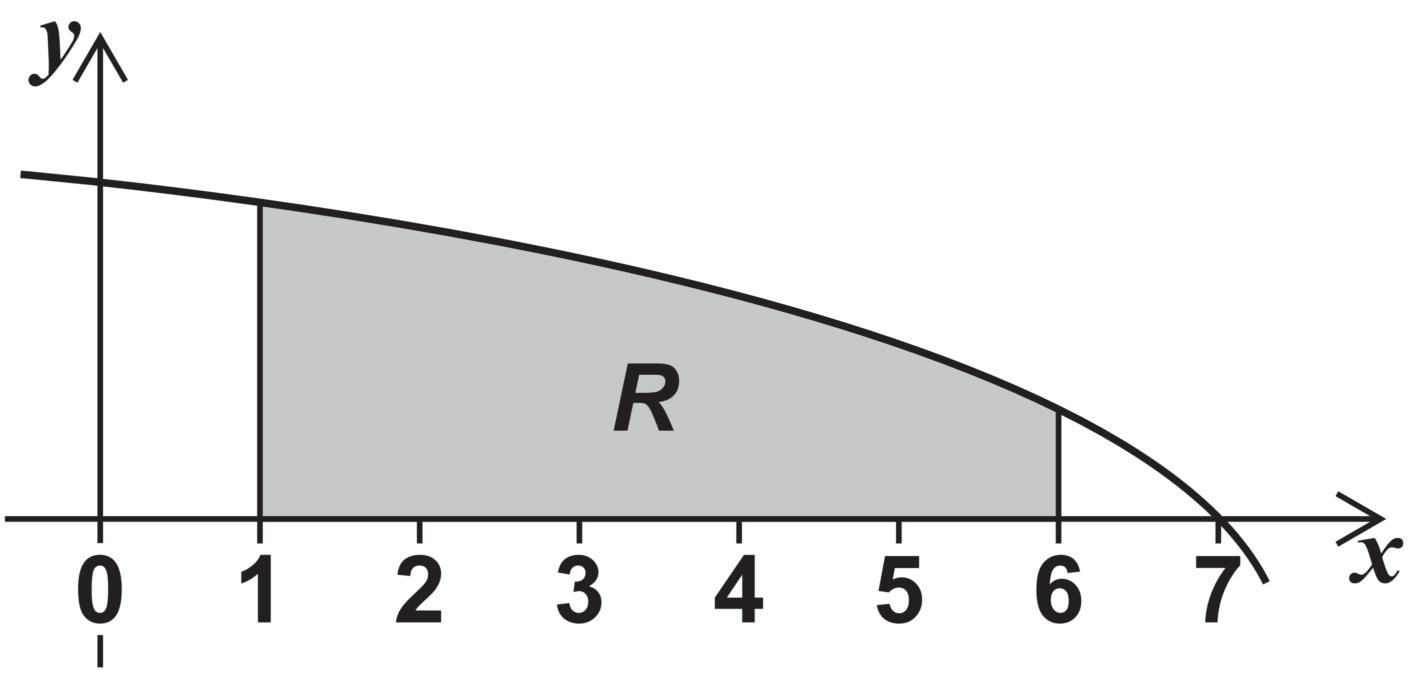
[Turn over]

11 The region R enclosed by the lines $x = 1$, $x = 6$, $y = 0$ and the curve

$$y = \ln(8 - x)$$

is shown shaded in FIGURE 3 below.

FIGURE 3



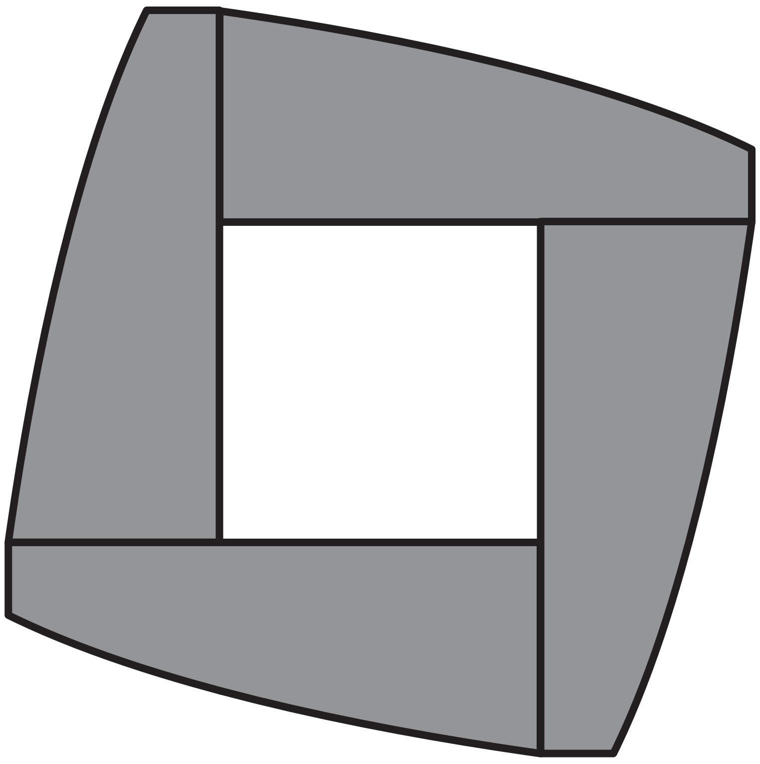
All distances are measured in centimetres.

11 (a) Use a single trapezium to find an approximate value of the area of the shaded region, giving your answer in cm^2 to two decimal places. [2 marks]

[Turn over]

11 (b) **Shape *B* is made from four copies of region *R* as shown in FIGURE 4 below.**

FIGURE 4



Shape *B* is cut from metal of thickness 2 mm

The metal has a density of 10.5 g/cm^3

Use the trapezium rule with SIX ordinates to calculate an approximate value of the mass of Shape *B*.

Give your answer to the nearest gram. [5 marks]

[Turn over]

11 (c) **Without further calculation, give one reason why the mass found in part (b) may be:**

11 (c) (i) an underestimate. [1 mark]

[Turn over]

11 (c) (ii) an overestimate. [1 mark]

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[Turn over]

12 **A curve *C* has equation**

$$x^3 \sin y + \cos y = Ax$$

where A is a constant.

***C* passes through the point $P\left(\sqrt{3}, \frac{\pi}{6}\right)$**

12 (a) **Show that $A = 2$ [2 marks]**

12(b) (i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2 - 3x^2 \sin y}{x^3 \cos y - \sin y}$

[5 marks]

[Turn over]

[Turn over]

12 (b) (ii) Hence, find the gradient of the curve at *P*. [2 marks]

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[Turn over]

12 (b) (iii) The tangent to *C* at *P* intersects the x-axis at *Q*.

Find the exact x-coordinate of *Q*. [4 marks]

[Turn over]

13 The function f is defined by

$$\mathrm{f}(x) = \frac{2x + 3}{x - 2} \qquad x \in \mathbb{R},\ x \neq 2$$

13 (a) (i) Find f^{-1} [3 marks]

13 (a) (ii) Write down an expression for $\mathrm{ff}(x)$. [1 mark]

13 (b) **The function g is defined by**

$$g(x) = \frac{2x^2 - 5x}{2} \qquad x \in \mathbb{R},\ 0 \leq x \leq 4$$

13 (b) (i) Find the range of g. [3 marks]

[Turn over]

13 (b) (ii) Determine whether g has an inverse.

Fully justify your answer.
[2 marks]

13 (c) **Show that**

$$\mathrm{gf}(x) = \frac{48 + 29x - 2x^2}{2x^2 - 8x + 8}$$

[4 marks]

[Turn over]

13 (d) It can be shown that fg is given by

$$\mathrm{fg}(x) = \frac{4x^2 - 10x + 6}{2x^2 - 5x - 4}$$

with domain

$\{x \in \mathbb{R} : 0 \leq x \leq 4, x \neq a\}$

Find the value of a.

Fully justify your answer.
[2 marks]

[Turn over]

14 **The function f is defined by**

$$\mathrm{f}(x) = 3^x\sqrt{x} - 1 \qquad \text{where } x \geq 0$$

14 (a) **$\mathrm{f}(x) = 0$ has a single solution at the point $x = \alpha$**

By considering a suitable change of sign, show that α lies between 0 and 1 [2 marks]

[Turn over]

14(b) (i) Show that

$$f'(x) = \frac{3^x(1 + x\ln 9)}{2\sqrt{x}}$$

[3 marks]

[Turn over]

14 (b) (ii) Use the Newton–Raphson method with $x_1 = 1$ to find x_3, an approximation for α.

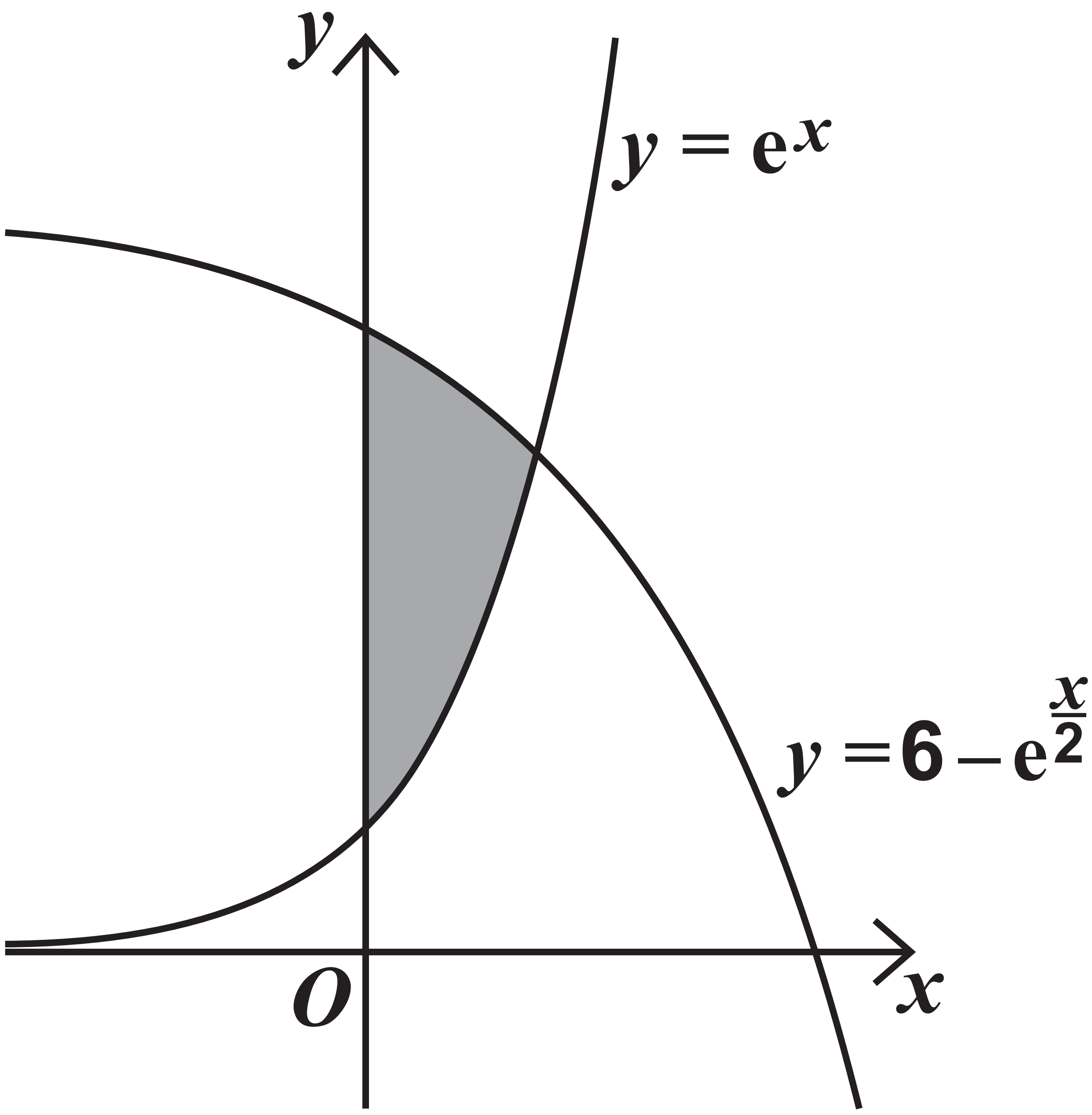
Give your answer to five decimal places. [2 marks]

[Turn over]

14 (b) (iii) Explain why the Newton–Raphson method fails to find α with $x_1 = 0$
[2 marks]

[Turn over]

15 **The region enclosed between the curves $y = e^x$, $y = 6 - e^{\frac{x}{2}}$ and the line $x = 0$ is shown shaded in the diagram below.**



Show that the exact area of the shaded region is

$6 \ln 4 - 5$

Fully justify your answer.
[10 marks]

[Turn over]

[Turn over]

END OF QUESTIONS

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