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AQA

Surname _____

Other Names _____

Centre Number _____

Candidate Number _____

Candidate Signature _____

A-level
MATHEMATICS

Paper 2

7357/2

Wednesday 10 June 2020 Afternoon

Time allowed: 2 hours

- **You must have the AQA Formulae for A-level Mathematics booklet.**
- **You should have a graphical or scientific calculator that meets the requirements of the specification.**

At the top of the page, write your surname and other names, your centre number, your candidate number and add your signature.

[Turn over]

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INSTRUCTIONS

- **Use black ink or black ball-point pen. Pencil should only be used for drawing.**
- **Answer ALL questions.**
- **You must answer each question in the space provided for that question.**
- **Do NOT write on blank pages.**
- **Show all necessary working; otherwise marks for method may be lost.**
- **Do all rough work in this book. Cross through any work that you do not want to be marked.**

INFORMATION

- **The marks for questions are shown in brackets.**
- **The maximum mark for this paper is 100.**

ADVICE

- **Unless stated otherwise, you may quote formulae, without proof, from the booklet.**
- **You do not necessarily need to use all the space provided.**

DO NOT TURN OVER UNTIL TOLD TO DO SO

SECTION A

Answer ALL questions in the spaces provided.

1 **Which one of these functions is decreasing for all real values of x?**

Circle your answer. [1 mark]

$\mathrm{f}(x) = \mathrm{e}^{x}$

$\mathrm{f}(x) = -\mathrm{e}^{1-x}$

$\mathrm{f}(x) = -\mathrm{e}^{x-1}$

$\mathrm{f}(x) = -\mathrm{e}^{-x}$

2 **Which one of the following equations has no real solutions?**

Tick (✓) ONE box. [1 mark]

☐ $\cot x = 0$

☐ $\ln x = 0$

☐ $|x + 1| = 0$

☐ $\sec x = 0$

3 Find the coefficient of x^2 in the binomial expansion of $\left(2x - \frac{3}{x}\right)^8$ [3 marks]

[Turn over]

4 **Using small angle approximations, show that for small, non-zero, values of x**

$$\frac{x \tan 5x}{\cos 4x - 1} \approx A$$

where A is a constant to be determined.
[4 marks]

[Turn over]

5 **Use integration by substitution to show that**

$$\int_{-\frac{1}{4}}^{6} x\sqrt{4x+1}\,\mathrm{d}x = \frac{875}{12}$$

Fully justify your answer. [6 marks]

[Turn over]

6 The line L has equation

$$5y + 12x = 298$$

A circle, C, has centre (7, 9)

L is a tangent to C.

6 (a) Find the coordinates of the point of intersection of L and C.

Fully justify your answer. [5 marks]

6 (b) **Find the equation of *C*. [3 marks]**

[Turn over]

7 a and b are two positive irrational numbers.

The sum of a and b is rational.

The product of a and b is rational.

Caroline is trying to prove $\frac{1}{a} + \frac{1}{b}$ is rational.

Here is her proof:

Step 1 $\frac{1}{a} + \frac{1}{b} = \frac{2}{a+b}$

Step 2 2 is rational and $a + b$ is non-zero and rational.

Step 3 Therefore $\frac{2}{a+b}$ is rational.

Step 4 Hence $\frac{1}{a} + \frac{1}{b}$ is rational.

7 (a) (i) Identify Caroline's mistake. [1 mark]

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[Turn over]

7 (a) (ii) Write down a correct version of the proof.
[2 marks]

7 (b) **Prove by contradiction that the difference of any rational number and any irrational number is irrational. [4 marks]**

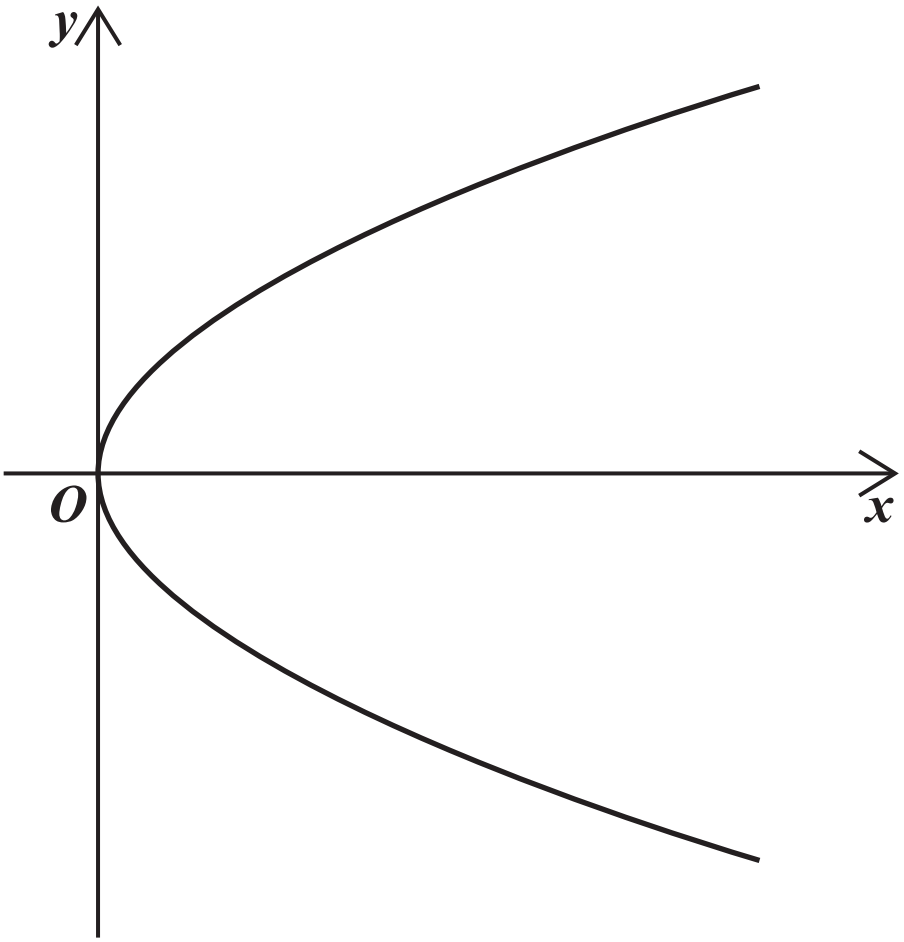
[Turn over]

8 The curve defined by the parametric equations

$x = t^2$ and $y = 2t$ $\quad -\sqrt{2} \leq t \leq \sqrt{2}$

is shown in FIGURE 1 below.

FIGURE 1



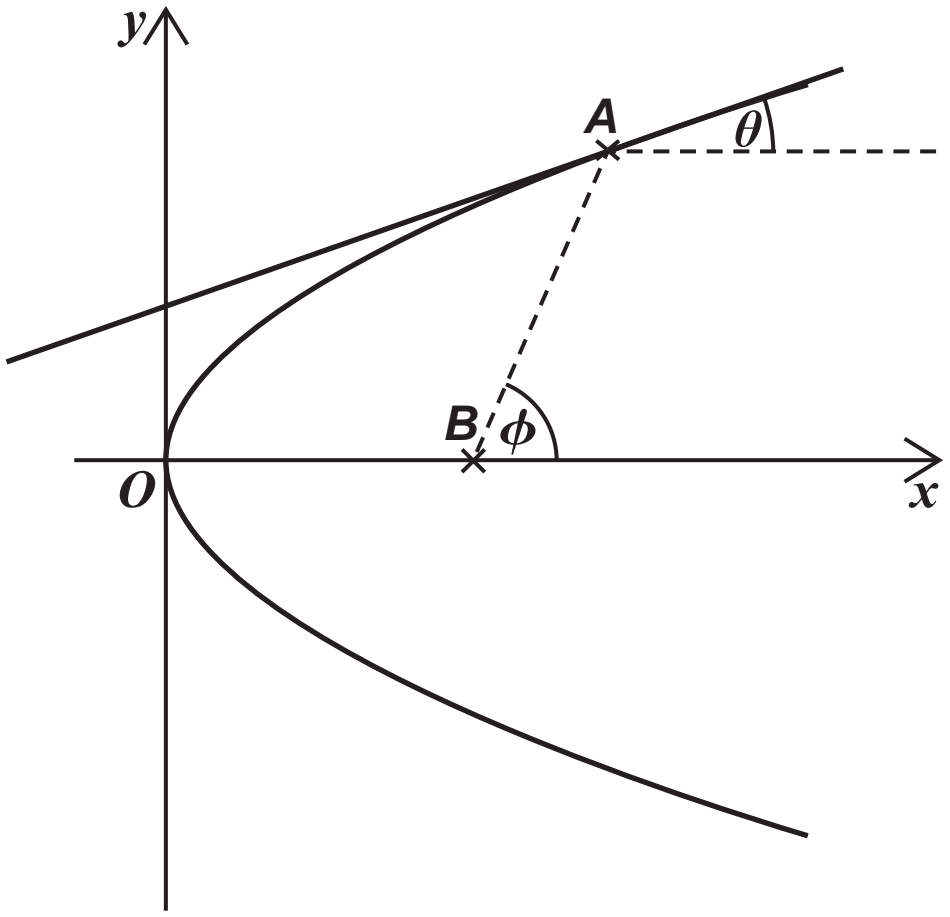
8 (a) **Find a Cartesian equation of the curve in the form $y^2 = \mathrm{f}(x)$ [2 marks]**

8 (b) The point *A* lies on the curve where $t = a$

The tangent to the curve at *A* is at an angle θ to a line through *A* parallel to the x-axis.

The point *B* has coordinates (1, 0)

The line *AB* is at an angle ϕ to the x-axis.



8 (b) (i) By considering the gradient of the curve, show that

$$\tan\theta = \frac{1}{a}$$

[3 marks]

8 (b) (ii) Find $\tan \phi$ in terms of a. [2 marks]

8 (b)(iii) Show that $\tan 2\theta = \tan \phi$ [3 marks]

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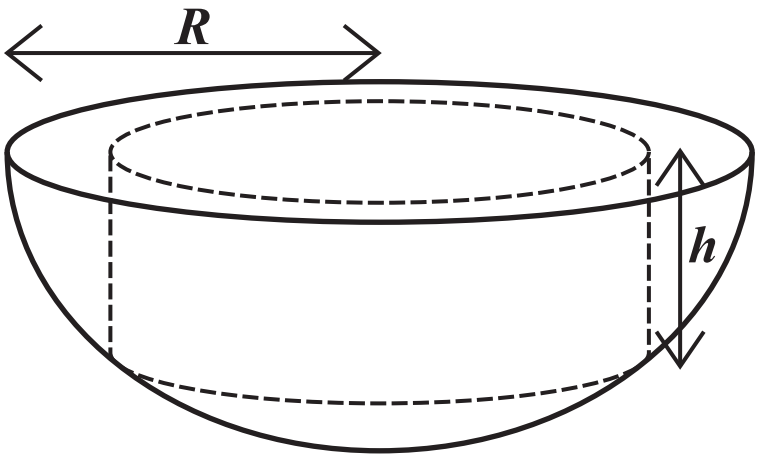
9 **A cylinder is to be cut out of the circular face of a solid hemisphere.**

The cylinder and the hemisphere have the same axis of symmetry.

The cylinder has height h and the hemisphere has a radius of R.



9 (a) **Show that the volume, V, of the cylinder is given by**

$$V = \pi R^2 h - \pi h^3$$

[3 marks]

9 (b) **Find the maximum volume of the cylinder in terms of R.**

Fully justify your answer. [7 marks]

[Turn over for Section B]

SECTION B

Answer ALL questions in the spaces provided.

10 A vehicle is driven at a constant speed of $12\,m\,s^{-1}$ along a straight horizontal road.

Only one of the statements below is correct.

Identify the correct statement.

Tick (✓) ONE box. [1 mark]

- ☐ The vehicle is accelerating
- ☐ The vehicle's driving force exceeds the total force resisting its motion
- ☐ The resultant force acting on the vehicle is zero
- ☐ The resultant force acting on the vehicle is dependent on its mass

11 A number of forces act on a particle such that the resultant force is $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$ N

One of the forces acting on the particle is $\begin{pmatrix} 8 \\ -5 \end{pmatrix}$ N

Calculate the total of the other forces acting on the particle.

Circle your answer. [1 mark]

$\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ N

$\begin{pmatrix} 14 \\ -8 \end{pmatrix}$ N

$\begin{pmatrix} -2 \\ 2 \end{pmatrix}$ N

$\begin{pmatrix} -14 \\ 8 \end{pmatrix}$ N

12 A particle, *P*, is moving with constant velocity $8\mathbf{i} - 12\mathbf{j}$

A second particle, *Q*, is moving with constant velocity $a\mathbf{i} + 9\mathbf{j}$

Q travels in a direction which is parallel to the motion of *P*.

Find a.

Circle your answer. [1 mark]

-6 -5 5 6

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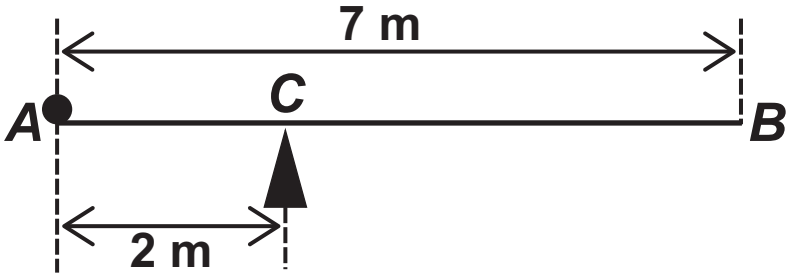
[Turn over]

13 A uniform rod, *AB*, has length 7 metres and mass 4 kilograms.

The rod rests on a single fixed pivot point, *C*, where $AC = 2$ metres.

A particle of weight W newtons is fixed at *A*, as shown in the diagram.



The system is in equilibrium with the rod resting horizontally.

13 (a) Find W, giving your answer in terms of g.
[2 marks]

13 (b) **Explain how you have used the fact that the rod is uniform in part (a). [1 mark]**

14 At time t seconds a particle, P, has position vector **r** metres, with respect to a fixed origin, such that

$$\mathbf{r} = (t^3 - 5t^2)\mathbf{i} + (8t - t^2)\mathbf{j}$$

14 (a) Find the exact speed of P when $t = 2$
[4 marks]

[Turn over]

14 (b) **Bella claims that the magnitude of acceleration of *P* will never be zero.**

Determine whether Bella's claim is correct.

Fully justify your answer. [3 marks]

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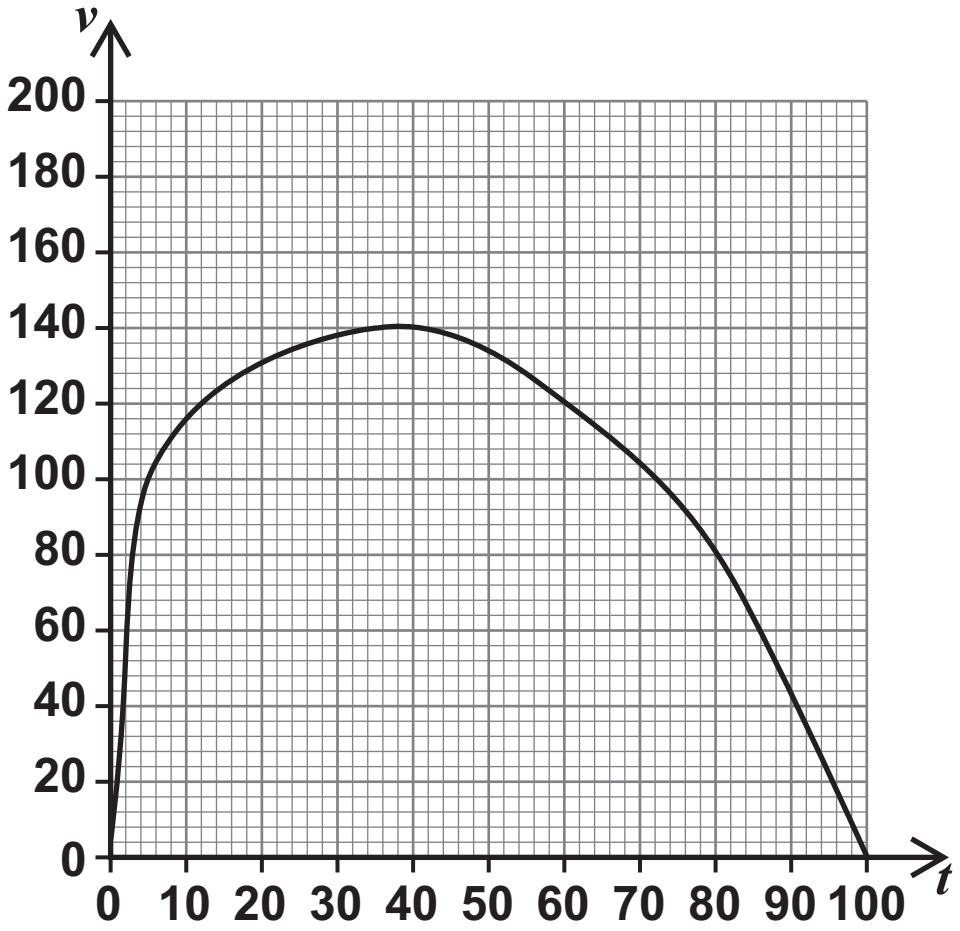
[Turn over]

15 A particle is moving in a straight line with velocity v m s^{-1} at time t seconds as shown by the graph below.



15 (a) Use the trapezium rule with four strips to estimate the distance travelled by the particle during the time period $20 \leq t \leq 100$ [4 marks]

[Turn over]

15 (b) **Over the same time period, the curve can be very closely modelled by a particular quadratic.**

Explain how you could find an alternative estimate using this quadratic. [1 mark]

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[Turn over]

16 **Two particles *A* and *B* are released from rest from different starting points above a horizontal surface.**

***A* is released from a height of h metres.**

***B* is released at a time t seconds after *A* from a height of kh metres, where $0 < k < 1$**

Both particles land on the surface 5 seconds after *A* was released.

Assuming any resistance forces may be ignored, prove that

$$t = 5(1 - \sqrt{k})$$

Fully justify your answer. [5 marks]

[Turn over]

17 A ball is projected forward from a fixed point, *P*, on a horizontal surface with an initial speed $u\,\text{m s}^{-1}$, at an acute angle θ above the horizontal.

The ball needs to first land at a point at least d metres away from *P*.

You may assume the ball may be modelled as a particle and that air resistance may be ignored.

Show that

$$\sin 2\theta \geq \frac{dg}{u^2}$$

[6 marks]

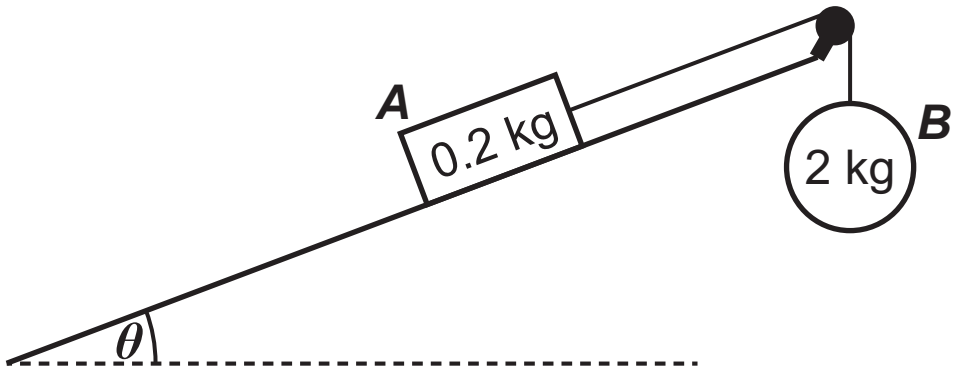
[Turn over]

18 Block *A*, of mass 0.2 kg, lies at rest on a rough plane.

The plane is inclined at an angle θ to the horizontal, such that $\tan\theta = \frac{7}{24}$

A light inextensible string is attached to *A* and runs parallel to the line of greatest slope until it passes over a smooth fixed pulley at the top of the slope.

The other end of this string is attached to particle *B*, of mass 2 kg, which is held at rest so that the string is taut, as shown in the diagram below.



18 (a) *B* is released from rest so that it begins to move vertically downwards with an acceleration of $\frac{543}{625}\,g\,\text{m s}^{-2}$

Show that the coefficient of friction between *A* and the surface of the inclined plane is 0.17
[8 marks]

18 (b) **IN THIS QUESTION USE $g = 9.81\,\mathrm{M\,S^{-2}}$**

When *A* reaches a speed of $0.5\,\mathrm{m\,s^{-1}}$ the string breaks.

18 (b) (i) **Find the distance travelled by *A* after the string breaks until first coming to rest. [4 marks]**

18 (b) (ii) State an assumption that could affect the validity of your answer to part (b)(i). [1 mark]

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[Turn over]

19 A particle moves so that its acceleration, $a\,\text{m s}^{-2}$, at time t seconds may be modelled in terms of its velocity, $v\,\text{m s}^{-1}$, as

$$a = -0.1v^2$$

The initial velocity of the particle is $4\,\text{m s}^{-1}$

19 (a) By first forming a suitable differential equation, show that

$$v = \frac{20}{5 + 2t}$$

[6 marks]

[Turn over]

19 (b) **Find the acceleration of the particle when $t = 5.5$ [2 marks]**

END OF QUESTIONS

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For Examiner's Use	
Question	Mark
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