

# Formulae for A-level Mathematics

# AS Mathematics (7356) A-level Mathematics (7357)

v1.2 First issued August 2017

For the new specifications for first teaching from September 2017.

This booklet of formulae is required for all AS and A-level Mathematics exams.

There is a larger booklet of formulae and statistical tables for all AS and A-level Further Mathematics exams.

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Further copies of this booklet are available from: Telephone: 0844 209 6614 Fax: 01483 452819 or download from the AQA website www.aqa.org.uk

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#### **Pure mathematics**

#### **Binomial series**

$$(a+b)^{n} = a^{n} + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^{2} + \dots + \binom{n}{r}a^{n-r}b^{r} + \dots + b^{n} \qquad (n \in \mathbb{N})$$
where  $\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$ 

$$(1+x)^{n} = 1 + nx + \frac{n(n-1)}{1.2}x^{2} + \dots + \frac{n(n-1)\dots(n-r+1)}{1.2\dots r}x^{r} + \dots \quad (|x| < 1, \ n \in \mathbb{Q})$$

#### **Arithmetic series**

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n[2a+(n-1)d]$$

#### Geometric series

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_{\infty} = \frac{a}{1 - r} \text{ for } |r| < 1$$

## Trigonometry: small angles

For small angle  $\theta$ , measured in radians:

$$\sin\theta \approx \theta$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$\tan\theta \approx \theta$$

## Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \frac{1}{2})\pi)$$

#### **Differentiation**

$$f(x)$$
  $f'(x)$ 

$$tan x$$
  $sec^2 x$ 

$$\csc x$$
  $-\csc x \cot x$ 

$$\sec x$$
  $\sec x \tan x$ 

$$\cot x$$
  $-\csc^2 x$ 

$$\frac{f(x)}{g(x)} \qquad \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

# Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

# Integration

$$\int u \frac{\mathrm{d}v}{\mathrm{d}x} \, \mathrm{d}x = uv - \int v \frac{\mathrm{d}u}{\mathrm{d}x} \, \mathrm{d}x$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$f(x)$$
  $\int f(x) dx$ 

$$tan x$$
  $ln|sec x| + c$ 

$$\cot x$$
  $\ln|\sin x| + c$ 

# **Numerical solution of equations**

The Newton-Raphson iteration for solving 
$$f(x) = 0$$
:  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ 

# **Numerical integration**

The trapezium rule: 
$$\int_a^b y \, dx \approx \frac{1}{2} h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$$
, where  $h = \frac{b-a}{n}$ 

#### **Mechanics**

## **Constant acceleration**

$$s = ut + \frac{1}{2}at^2$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$s = vt - \frac{1}{2}at^2$$
  $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ 

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

$$v = u + at$$

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$s = \frac{1}{2}(u+v)t$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$v^2 = u^2 + 2as$$

# **Probability and statistics**

# **Probability**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) \times P(B|A)$$

#### Standard deviation

$$\sqrt{\frac{\sum (x - \overline{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \overline{x}^2}$$

# **Discrete distributions**

Distribution of X	P(X = x)	Mean	Variance
Binomial $B(n, p)$	$\binom{n}{x}p^x(1-p)^{n-x}$	пр	np(1-p)

# Sampling distributions

For a random sample of *n* observations from  $N(\mu, \sigma^2)$ :

$$\frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1)$$

#### **End of formulae**

