GCE A2

Mathematics

January 2009

Mark Schemes

Issued:April 2009

NORTHERN IRELAND GENERAL CERTIFICATE OF SECONDARY EDUCATION (GCSE) AND NORTHERN IRELAND GENERAL CERTIFICATE OF EDUCATION (GCE)

MARK SCHEMES (2009)

Foreword

Introduction

Mark Schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of 16- and 18-year-old students in schools and colleges. The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes therefore are regarded as a part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

The Council hopes that the mark schemes will be viewed and used in a constructive way as a further support to the teaching and learning processes.

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General Certificate of Education
January 2009

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MARK
SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

Introduction

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The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

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The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

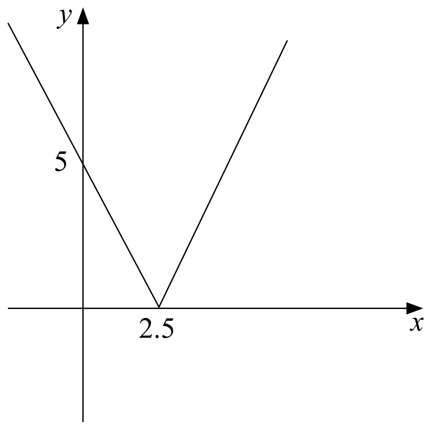
(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

AVAILABLE MARKS

1 **(i)**



M1W1

(ii) $2x - 5 > 10$ or $-(2x - 5) > 10$ MW1

$2x > 15$ $\quad -2x + 5 > 10$ W1

$x > 7.5$ $\quad x < -\frac{5}{2}$ MW2

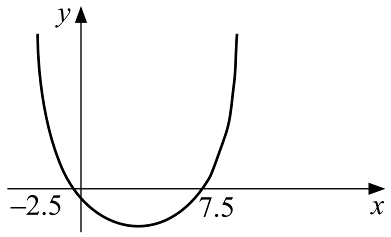
Alternative Solution

(ii) $(2x - 5)^2 > 10^2$ M1

$4x^2 - 20x - 75 > 0$

$(2x - 15)(2x + 5) = 0$

$x = 7.5$ or $x = -2.5$ MW1



$x > 7.5$ or $x < -2.5$ MW2 6

2

x	y
0	1.73205
0.5	1.69634
1	1.59383
1.5	1.43900
2	1.25851
2.5	1.09492
3	1.00499

M1MW2

$$A \approx \frac{1}{3}h[y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) + y_6]$$

$$= \frac{1}{3} \times \frac{1}{2}\,[1.73205 + 4(4.23027) + 2(2.85234) + 1.00499]$$

M1MW1

$= 4.22713 \approx 4.23$ W1 6

				AVAILABLE MARKS
3	**(i)**	$\cos\theta = \frac{x-3}{3}$ and $\sin\theta = \frac{y}{2}$	M1W1	
		$\sin^2\theta + \cos^2\theta = 1$		
		$\left(\frac{x-3}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$	M1W1	
	(ii)	$y = 0 \Rightarrow \left(\frac{x-3}{3}\right)^2 = 1$	M1	
		$x^2 - 6x + 9 = 9$	MW1	
		$x(x-6) = 0$		
		$x = 0$ or $x = 6$	MW1	
		(0,0) or (6,0)		
		Alternative Solution		
	(ii)	$y = \Rightarrow 2\sin\theta = 0 \Rightarrow \theta = 0°$ or $\theta = 180°$	MW1	
		$\theta = 0° \Rightarrow x = 6$	MW1	
		$\theta = 180° \Rightarrow x = 0$	MW1	7
		(0,0) or (6,0)		
4	**(a)**	$\frac{4x^2-9}{x^2+2x+1} \times \frac{2x+2}{2x-3}$	M1	
		$\frac{(2x-3)(2x+3)}{(x+1)(x+1)} \times \frac{2(x+1)}{(2x-3)}$	M1W2	
		$\frac{2(2x+3)}{x+1}$	MW1	
	(b)	$x^2 - 2x\overset{\displaystyle 2}{\overline{)2x^2 - 9}}$		
		$\frac{2x^2 - 4x}{4x - 9}$	M1W1	
		$2 + \frac{4x-9}{x(x-2)}$		
		$\frac{4x-9}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2}$	M1W1	
		$4x - 9 = A(x-2) + Bx$	M1	
		Let $x = 2$ $8 - 9 = 2B$	M1	
		$B = -\frac{1}{2}$	W1	
		Let $x = 0$ $0 - 9 = -2A$		
		$A = \frac{9}{2}$	MW1	
		$\frac{2x^2-9}{x(x-2)} = 2 + \frac{9}{2x} - \frac{1}{2(x-2)}$	MW1	14

				AVAILABLE MARKS
5	**(i)**	$C = 30°$	MW1	
	(ii)	$130 = 180 - 150\mathrm{e}^{5k}$	M1	
		$\mathrm{e}^{5k} = \frac{1}{3}$		
		$5k = \ln\frac{1}{3}$	M1W1	
		$k = -0.219722 \approx -0.22$	W1	
	(iii)	$\frac{\mathrm{d}C}{\mathrm{d}t} = 33\mathrm{e}^{-0.22t}$	M1W1	
		$t = 10 \Rightarrow \frac{\mathrm{d}C}{\mathrm{d}t} = 3.6565 \approx 3.66$	M1W1	9
6	**(a) (i)**	$u = 5x \quad v = \ln(x^2 - 2)$		
		$\frac{\mathrm{d}u}{\mathrm{d}x} = 5 \quad \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{2x}{x^2 - 2}$	MW2	
		$\frac{\mathrm{d}y}{\mathrm{d}x} = 5x\left(\frac{2x}{x^2 - 2}\right) + 5\ \ln(x^2 - 2)$	M1W1	
		$\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\frac{10x^2}{x^2 - 2}\right) + 5\ \ln(x^2 - 2)$		
	(ii)	$u = \sin x \quad v = \cos 3x$		
		$\frac{\mathrm{d}u}{\mathrm{d}x} = \cos x \quad \frac{\mathrm{d}v}{\mathrm{d}x} = -3\sin 3x$		
		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos 3x(\cos x) - \sin x(-3\sin 3x)}{\cos^2 3x}$		
		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos 3x \cos x + 3\sin 3x \sin x}{\cos^2 3x}$	M1W3	
	(b)	$2x + 2\ \ln x + \frac{1}{3}\mathrm{e}^{3x} + \tan x + c$	MW5	13
7		$\sqrt{4 - x} = (4 - x)^{\frac{1}{2}}$	MW1	
		$(4 - x)^{\frac{1}{2}} = \left[4\left(1 - \frac{x}{4}\right)\right]^{\frac{1}{2}} = 2\left(1 - \frac{x}{4}\right)^{\frac{1}{2}}$	M1W1	
		$2\left(1 - \frac{x}{4}\right)^{\frac{1}{2}} = 2\left[1 + \frac{1}{2}\left(\frac{-x}{4}\right) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(\frac{-x}{4}\right)^2}{2} + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(\frac{-x}{4}\right)^3}{2 \times 3}\right]$	MW3	
		$2\left[1 - \frac{x}{8} - \frac{x^2}{128} - \frac{x^3}{1024}\right] = 2 - \frac{x}{4} - \frac{x^2}{64} - \frac{x^3}{512}$	MW1	7

AVAILABLE MARKS

8 **(a)** $\tan^2\theta + 2(\tan^2\theta + 1) = 11$ M1W1

$3\tan^2\theta = 9$

$\tan\theta = \sqrt{3}$ or $\tan\theta = -\sqrt{3}$ MW2

$\theta = \frac{\pi}{3}, \frac{-2\pi}{3}$ or $\frac{2\pi}{3}, \frac{-\pi}{3}$ MW2

(b) LHS $\frac{\sec\theta - \cos\theta}{\operatorname{cosec}\theta - \sin\theta}$

$$\frac{\frac{1}{\cos\theta} - \cos\theta}{\frac{1}{\sin\theta} - \sin\theta}$$ M1W1

$$\frac{\frac{1-\cos^2\theta}{\cos\theta}}{\frac{1-\sin^2\theta}{\sin\theta}}$$ M1W1

$$\frac{\frac{\sin^2\theta}{\cos\theta}}{\frac{\cos^2\theta}{\sin\theta}}$$ M1W1

$$\frac{\sin^2\theta}{\cos\theta} \times \frac{\sin\theta}{\cos^2\theta} \equiv \frac{\sin^3\theta}{\cos^3\theta}$$

$\tan^3\theta \equiv$ RHS MW1

Alternative Solution

LHS $$\frac{\frac{1}{\cos\theta} - \cos\theta}{\frac{1}{\sin\theta} - \sin\theta} \times \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$$ M2W2

$$\frac{\sin\theta - \sin\theta\cos^2\theta}{\cos\theta - \sin^2\theta\cos\theta}$$

$$\frac{\sin\theta(1-\cos^2\theta)}{\cos\theta(1-\sin^2\theta)} \equiv \frac{\sin^3\theta}{\cos^3\theta}$$ M1W1

$\tan^3\theta \equiv$ RHS MW1 13

Total **75**

ADVANCED
General Certificate of Education
January 2009

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

WEDNESDAY 21 JANUARY, AFTERNOON

MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

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When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

			AVAILABLE MARKS
1	direction $\mathbf{r}_1$ is given by $(3\mathbf{i} + 4\mathbf{j} - \mathbf{k})$	M1	
	$\mathbf{r}_2$ $(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$		
	$\mathbf{a} \cdot \mathbf{b} = \lvert\mathbf{a}\rvert\ \lvert\mathbf{b}\rvert \cos\theta$	M1	
	$\mathbf{a}\cdot\mathbf{b} = \sqrt{3^2 + 4^2 + 1^2}\ \ \sqrt{1^2 + 1^2 + 2^2}\cos\theta$	M1	
	$-3 = \sqrt{26}\ \ \sqrt{6}\cos\theta$	M1W1W2	
	$\cos\theta = \dfrac{-3}{\sqrt{156}}$		
	$\theta = 104°$	W1	8
2	**(i)** $x = 2t \qquad y = t^3 - 3t$		
	$\dfrac{dx}{dt} = 2 \qquad \dfrac{dy}{dt} = 3t^2 - 3$	MW2	
	$\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$	M1	
	$= \dfrac{3t^2 - 3}{2}$	W1	
	(ii) $\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{dy}{dy}\right) \times \dfrac{dt}{dx}$	M1	
	$= 3t \times \dfrac{1}{2}$		
	$= \dfrac{3t}{2}$	W2	7

AVAILABLE MARKS

3 $\frac{dx}{dt} = \frac{k}{x}$

$\int x \, dx = \int k \, dt$ M2W1

$\frac{x^2}{2} = kt + c$ MW2

when $x = 100$, $t = 0$ M1

$\frac{10000}{2} = c \Rightarrow c = 5000$ W1

when $x = 50$, $t = 5$

$\frac{2500}{2} = 5k + 5000$

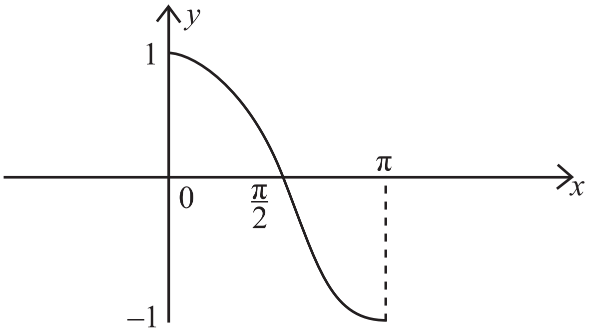
$-750 = k$ MW1

$\frac{x^2}{2} = -750t + 5000$

when $x = 0$, $t = \frac{5000}{750}$ M1

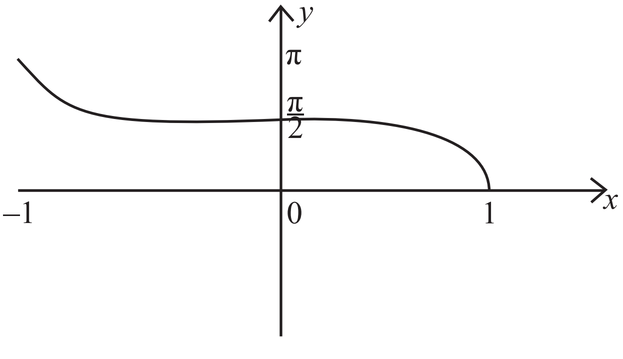
$= \frac{20}{3}$ s W1 10

4 **(i)**



MW2

(ii)



MW3

5

				AVAILABLE MARKS
5	**(i)**	$\frac{3x+4}{x(x+1)} = \frac{\mathrm{A}}{x} + \frac{\mathrm{B}}{x+1}$	M1W1	
		$3x+4 = \mathrm{A}(x+1) + \mathrm{B}x$	M1	
		Let $x = -1$ $1 = -\mathrm{B} \Rightarrow \mathrm{B} = -1$	M1W1	
		$x = 0$ $4 = \mathrm{A}$	W1	
		$\frac{3x+4}{x(x+1)} = \frac{4}{x} - \frac{1}{x+1}$		
	(ii)	Area $= \int_2^3 \frac{3x+4}{x(x+1)}\,\mathrm{d}x$	M1W1	
		$= \int_2^3 \frac{4}{x} - \frac{1}{x+1}\,\mathrm{d}x$	MW1	
		$= [4 \ln x \ln\lvert x+1\rvert]_2^3$	W2	
		$= (4 \ln 3 - \ln 4) - (4 \ln 2 - \ln 3)$	M1	
		$= 5 \ln 3 - 6 \ln 2$		
		$= \ln \frac{243}{64}$	W1	13
6	**(i)**	$\mathrm{f}(x) \geqslant 5$	MW1	
	(ii)	fg $x \to \lvert x\rvert \to 2\lvert x\rvert + 5$	M1W1	
		fg: $x \to 2\lvert x\rvert + 5$	MW1	
		domain $x > -1$	MW1	
	(iii)	$y = 2x + 5$	M1	
		$\frac{y-5}{2} = x$	W1	
		$\mathrm{f}^{-1} : x \to \frac{x-5}{2}$	MW1	
		domain $x > 5$	MW1	
		range f^{-1} $\mathrm{f}^{-1}(x) \geqslant 0$	MW1	10

				AVAILABLE MARKS
7	**(i)**	$\sin 3A = \sin(A + 2A)$	M1	
		$= \sin A \cos 2A + \cos A \sin 2A$	M1W1	
		$= \sin A(2\cos^2 A - 1) + 2\sin A\cos^2 A$	M1W1	
		$= 4\sin A\cos^2 A - \sin A$		
		$= 4\sin A(1 - \sin^2 A) - \sin A$	M1	
		$= 4\sin A - 4\sin^3 A - \sin A$		
		$= 3\sin A - 4\sin^3 A$	W1	
	(ii)	$\sin A + \sin 3A = 0$		
		$\sin A + 3\sin A - 4\sin^3 A = 0$		
		$\sin A - \sin^3 A = 0$	M1W1	
		$\sin A[1 - \sin^2 A] = 0$		
		$\sin A = 0$ or $\sin A = \pm 1$	MW3	
		$A = 0°, 90°, 180°, 270°, 360°$	MW3	15
8		$\int x \operatorname{cosec}^2 x \, dx$	M1	
		$= -x \cot x + \int \cot x \, dx$	W3	
		$= -x \cot x + \ln\lvert\sin x\rvert + c$	W3	7
			Total	**75**

ADVANCED
General Certificate of Education
January 2009

Mathematics

Assessment Unit F2
assessing
Module FP2: Further Pure Mathematics 2

[AMF21]

THURSDAY 29 JANUARY, MORNING

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Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1	$\tan\left(2\theta+\frac{\pi}{4}\right)=\tan\left(\frac{\pi}{3}-3\theta\right)$	M1W1	
	$2\theta+\frac{\pi}{4}=n\pi+\left(\frac{\pi}{3}-3\theta\right)$	M2W1	
	$5\theta=n\pi+\frac{\pi}{12}$		
	$\theta=n\frac{\pi}{5}+\frac{\pi}{60}$	W1	6

			AVAILABLE MARKS
2 **(i)**	$\frac{x^3-4x^2+9x+10}{(x^2+5)(x-3)^2}=\frac{Ax+B}{x^2+5}+\frac{C}{x-3}+\frac{D}{(x-3)^2}$	M1MW1	
	$x^3-4x^2+9x+10\ =(x-3)^2(Ax+B)+(x-3)(x^2+5)\ C+(x^2+5)D$	M1	
	$x=3\quad 27-36+27+10=14D$	M1	
	$D=2$	W1	
	Coeff of x^3: $A+C=1$		
	Coeff of x^2: $B-6A-3C+D=-4$		
	$B-6A-3C=-6$		
	$B+3C=0$		
	Coeff of x^0: $9B-15C+5D=10$		
	$9B=15C$		
	$\Rightarrow B=C=0$	W2	
	$\Rightarrow A=1$	W1	
	$\therefore$ exp r $\equiv\frac{x}{x^2+5}+\frac{2}{(x-3)^2}$		
(ii)	$(x^2+5)(x-3)\frac{\mathrm{d}y}{\mathrm{d}x}-(x^2+5)y=x^3-4x^2+9x+10$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}-\frac{1}{x-3}y=\frac{x^3-4x^2+9x+10}{(x^2+5)(x-3)}$	MW1	
	IF $=\mathrm{e}^{\int\frac{-1}{x-3}\mathrm{d}x}=\mathrm{e}^{-\ln(x-3)}=\frac{1}{x-3}$	M1W1	
	$\frac{1}{x-3}\frac{\mathrm{d}y}{\mathrm{d}x}-\frac{1}{(x-3)^2}y=\frac{x^3-4x^2+9x+10}{(x^2+5)(x-3)^2}$	MW1	
	$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1}{x-3}y\right]=\text{RHS}$	MW1	
	$=\frac{x}{x^2+5}+\frac{2}{(x-3)^2}$	M1	
	$\frac{y}{x-3}=\frac{1}{2}\ln(x^2+5)-\frac{2}{(x-3)}+C$	M1 W1	
	Put $x=4$, $y=-2$		
	$-2=\frac{1}{2}\ln 21-2+C\Rightarrow C=-\frac{1}{2}\ln 21$	M1W1	18
	$y=\frac{1}{2}(x-3)\ln\left(\frac{x^2+5}{21}\right)-2$		

3 **(i)**

$$\left.\begin{array}{llll} f(x) & = \ln(1+x) & f(0) & = 0 \\ f'(x) & = (1+x)^{-1} & f'(0) & = 1 \\ f''(x) & = -(1+x)^{-2} & f''(0) & = -1 \\ f'''(x) & = 2(1+x)^{-3} & f'''(0) & = 2 \\ f^{\text{iv}}(x) & = -6(1+x)^{-4} & f^{\text{iv}}(0) & = -6 \\ f^{\text{v}}(x) & = 24(1+x)^{-5} & f^{\text{v}}(0) & = 24 \end{array}\right\}$$ M1W2

$f(x) = f(0) + xf'(0) + x^2\dfrac{f''(0)}{2!} + x^3\dfrac{f'''(0)}{3!} + \ldots$ M1

$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 \ldots$ W1

(ii) $\ln\left[\dfrac{1+x}{1-x}\right] = \ln(1+x) - \ln(1-x)$ M1

$= \ln(1+x) - \left\{-x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 - \frac{1}{5}x^5\right\} \ldots.$

$= 2\left\{x + \frac{1}{3}x^3 + \frac{1}{5}x^5\right\} \ldots.$ W2

(iii) $\dfrac{1+x}{1-x} = 2 \qquad x = \dfrac{1}{3}$ M1

$\ln 2 \doteqdot 2\left\{\frac{1}{3} + \frac{1}{3}^{4} + \frac{1}{5}\frac{1}{3}^{5}\right\} \ldots.$ W1

$= 2\left\{\frac{1}{3} + \frac{1}{81} + \frac{1}{1215}\right\}$

$= \dfrac{842}{1215}$ W1 11

4 **(a)** $\left(\cos\frac{\pi}{7}+\mathrm{i}\sin\frac{\pi}{7}\right)^3 = \left(\mathrm{e}^{\mathrm{i}\frac{\pi}{7}}\right)^3 = \mathrm{e}^{\frac{3}{7}\mathrm{i}\pi}$ M1W1

$\left(\cos\frac{\pi}{7}-\mathrm{i}\sin\frac{\pi}{7}\right)^{-4} = \left(\mathrm{e}^{-\mathrm{i}\frac{\pi}{7}}\right)^{-4} = \mathrm{e}^{\frac{4}{7}\mathrm{i}\pi}$ W1

Product $= \mathrm{e}^{\frac{7}{7}\mathrm{i}\pi} = \mathrm{e}^{\mathrm{i}\pi} = -1$ MW1

(b) $z^4 = -4$

$(r\mathrm{e}^{\mathrm{i}\theta})^4 = 4\mathrm{e}^{\pi\mathrm{i} + 2n\pi\mathrm{i}}$

$r^4\mathrm{e}^{4\mathrm{i}\theta} = 4\mathrm{e}^{\mathrm{i}(\pi+2n\pi)}$ $n = 0,1,2,3 \ldots.$ M1

$\therefore\ r = \sqrt{2}\ \theta = \frac{\pi}{4} + \frac{n\pi}{2}$ $n = 0, 1, 2, 3 \ldots.$ MW2

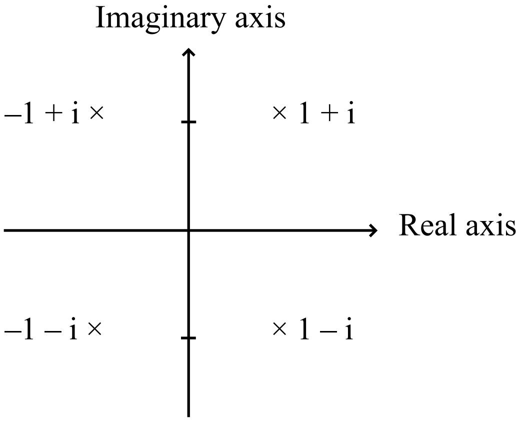
Roots $\sqrt{2}\,\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}$

$\sqrt{2}\left(\cos\frac{\pi}{4} = \mathrm{i}\sin\frac{\pi}{4}\right)$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}} + \mathrm{i}\,\frac{1}{\sqrt{2}}\right) = 1 + \mathrm{i}$

with 90° rotations MW1

$= \pm 1 \pm \mathrm{i}$



MW2 12

5 **(i)** When $n = 1$ $x^n = x$ and $\dfrac{x^n - 1}{x - 1} = 1$

so $\mathbf{A}^1 = \begin{pmatrix} x^1 & \dfrac{x^1 - 1}{x - 1} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} x & 1 \\ 0 & 1 \end{pmatrix}$ is true (1) MW1

Assume $\mathbf{A}^k = \begin{pmatrix} x^k & \dfrac{x^k - 1}{x - 1} \\ 0 & 1 \end{pmatrix}$ M1

Consider $\mathbf{A}^{k+1}$ M1

$= \mathbf{A}\,\mathbf{A}^k$

$= \begin{pmatrix} x & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x^k & \dfrac{x^k - 1}{x - 1} \\ 0 & 1 \end{pmatrix}$ MW1

$= \begin{pmatrix} x^{k+1} & \dfrac{x^{k+1} - x}{x - 1} + 1 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} x^{k+1} & \dfrac{x^{k+1} - 1}{x - 1} \\ 0 & 1 \end{pmatrix}$ W1

$\therefore$ statement true for $n = k + 1$ (2) M1

(1) and (2) imply statements true for all $n \geqslant 1$, $n \in \mathbb{Z}$ M1

(ii) $\mathbf{B}^{10} = \begin{pmatrix} 2^{10} & \dfrac{2^{10} - 1}{2 - 1} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1024 & 1023 \\ 0 & 1 \end{pmatrix}$ M1W1 9

AVAILABLE MARKS

6 **(i)** $a^2 = 4 \quad b^2 = a^2(1 - e^2) = 3$

$4 - 4e^2 = 3 \quad e = \frac{1}{2}$ M1W1

Focus $(ae, 0) = (\frac{1}{2} \times 2, 0) = (1, 0)$ MW1

Directrix $x = \frac{a}{e} = 2 \div \frac{1}{2} = 4$ MW1

(ii) $\frac{x^2}{4} + \frac{y^2}{3} = \frac{4\cos^2\theta}{4} + \frac{3\sin^2\theta}{3} = 1$ M1W1

(iii) At P, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}\theta}}{\frac{\mathrm{d}x}{\mathrm{d}\theta}}$ M1

$= \frac{\sqrt{3}\cos\theta}{-2\sin\theta}$ W2

$y - \sqrt{3}\sin\theta = \frac{\sqrt{3}\cos\theta}{2\sin\theta}\ (x - 2\cos\theta)$ M1W1

$\frac{y\sin\theta}{\sqrt{3}} - \sin^2\theta = -\frac{\cos\theta}{2}x + \cos^2\theta$

$\frac{x}{2}\cos\theta + \frac{y}{\sqrt{3}}\sin\theta = \cos^2\theta + \sin^2\theta = 1$ W1

(iv) On directrix $x = 4 \ \therefore\ \frac{y}{\sqrt{3}}\sin\theta = 1 - 2\cos\theta$ M1W1

$Q\left(4, \frac{\sqrt{3}(1 - 2\cos\theta)}{\sin\theta}\right)$ W1

$m_{\mathrm{PS}} = -\frac{\sqrt{3}\sin\theta}{1 - 2\cos\theta}$ M1W1

$m_{\mathrm{SQ}} = \frac{\sqrt{3}}{3}\frac{(1 - 2\cos\theta)}{\sin\theta}$ MW1

$m_{\mathrm{PS}} \times m_{\mathrm{SQ}} = -\frac{\sqrt{3}\sin\theta}{1 - 2\cos\theta}\frac{1 - 2\cos\theta}{\sqrt{3}\sin\theta} = -1$ MW1

$\therefore\ \mathrm{P}\hat{\mathrm{S}}\mathrm{Q} = 90°$ 19

Total **75**

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General Certificate of Education
January 2009

Mathematics

Assessment Unit M2
assessing
Module M2: Mechanics 2

[AMM21]

TUESDAY 27 JANUARY, AFTERNOON

MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method

W indicates marks for correct working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidates's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

					AVAILABLE MARKS
1	(i)	$\text{KE} = \frac{1}{2}mv^2$		M1	
		$= \frac{1}{2} \times 1500 \times 25$			
		$= 18\,750\,\text{J}$		W1	
		$= 18\,800\,\text{J}$			
	(ii)	Work done = change in KE		M1	
		$= \frac{1}{2} \times 1500 \times 0 - \frac{1}{2} \times 1500 \times 25$		W1	
		$= -18\,750\,\text{J}$		W1	
		$= -18\,800\,\text{J}$			
	(iii)	$\text{WD} = Fs$		M1	
		$-18\,750 = -30F$		W1	
		$F = 625\,\text{N}$		W1	8
2	(i)	Total force $= 4\mathbf{i} + \mathbf{j} - 2\mathbf{i} + \mathbf{j}$			
		$= 2\mathbf{i} + 2\mathbf{j}$		M1W1	
		$\mathbf{F} = m\mathbf{a}$		M1	
		$2\mathbf{i} + 2\mathbf{j} = 0.5\mathbf{a}$			
		$\mathbf{a} = 4\mathbf{i} + 4\mathbf{j}$		W1	
	(ii)	$\mathbf{u} = \mathbf{i} - \mathbf{j}$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$	M1	
		$\mathbf{v} = ?$	$\mathbf{v} = \mathbf{i} - \mathbf{j} + (4\mathbf{i} + 4\mathbf{j})$		
		$t = 1$	$\mathbf{v} = 5\mathbf{i} + 3\mathbf{j}$	W1	
		$\mathbf{a} = 4\mathbf{i} + 4\mathbf{j}$	$\lvert\mathbf{v}\rvert = \sqrt{25 + 9} = \sqrt{34}\,\text{m}\,\text{s}^{-1}$	M1W1	
	(iii)	$\mathbf{u} = \mathbf{i} - \mathbf{j}$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$	M1	
		$t = 4$	$\mathbf{s} = 36\mathbf{i} + 28\mathbf{j}$	W1	
		$\mathbf{a} = 4\mathbf{i} + 4\mathbf{j}$			10

				AVAILABLE MARKS
3	**(i)**	$\mathbf{r} = 3t^2\,\mathbf{i} + (2t^3 - t)\,\mathbf{j} + 2t\mathbf{k}$		
		$\mathbf{v} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = 6t\,\mathbf{i} + (6t^2 - 1)\,\mathbf{j} + 2\mathbf{k}$	M1W1	
	(ii)	Initial velocity when $t = 0$	M1	
		$\mathbf{v} = -\,\mathbf{j} + 2\mathbf{k}$	W1	
	(iii)	$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = 6\mathbf{i} + 12t\mathbf{j}$	M1W1	
		at $t = 3$		
		$\mathbf{a} = 6\mathbf{i} + 36\mathbf{j}$	MW1	
	(iv)	Magnitude of acceleration increasing	MW1	
		or no other value of t makes $\mathbf{r} = \mathbf{0}$		8
4	**(i)**	Power = rate of change of energy	M1	
		$= \frac{\frac{1}{2}mv^2 - \frac{1}{2}mu^2 + mgh}{t}$	M1	
		$= \frac{1}{2} \times 100 \times 20^2 - 0 + 100 \times 9.8 \times 8$	W2	
		= 27 800 watts	W1	
	(ii)	$u = 20$ $\quad v^2 = u^2 + 2as$	M1	
		$v = 0$ $\quad 0 = 400 - 19.6s$	M1	
		$a = -9.8$ $\quad s = 20.4\,\text{m}$	W1	
		$s = ?$		
	(iii)	no resistance to motion	MW1	9

5 **(i)** $\text{PE} = mgh$

$= -Mgd\sin 30° = \dfrac{-Mgd}{2}$

M1W2

(ii) $0 + \frac{1}{2}Mu^2 = -Mgd\sin 30° + \frac{1}{2}M(2u)^2$ M1W2

$\dfrac{Mgd}{2} = \dfrac{3}{2}Mu^2$

$u = \sqrt{\dfrac{gd}{3}}$ W1 7

6 **(i)** Vertical $s = ut + \frac{1}{2}at^2$ M1

$u = 0$ $19.6 = 4.9t^2$

$a = 9.8$ $t = 2\text{s}$ W1

$s = 19.6$

$t = ?$

(ii) Horizontal $s = ut + \frac{1}{2}at^2$ M1

$u = 20$ $= 2 \times 20$

$t = 2$ $= 40\,\text{m}$ W1

$a = 0$

$s = ?$

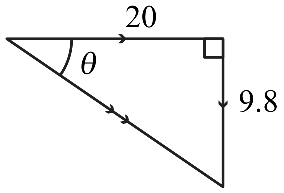
(iii) Vertical

$u = 0$ $v = u + at$ M1

$t = 1$ $v = 9.8\,\text{ms}^{-1}$ W1

$a = 9.8$

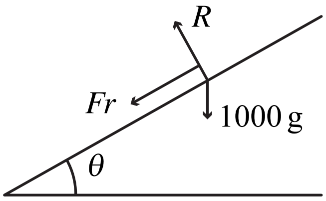
$v = ?$



M1

$\tan\theta = \dfrac{9.8}{20}$ M1

$\theta = 26.1°$ below horizontal W1 9

			AVAILABLE MARKS
7	**(i)** 	MW2	
	(ii) $F = ma$	M1	
	$\updownarrow R\cos\theta - 1000\text{g} - \mu R\sin\theta = 0$	W2	
	$R\frac{12}{13} - 9800 - 0.4R\frac{5}{13} = 0$	MW1	
	$R = 12\,740\text{ N}$		
	$12\,700\text{ N}$	W1	
	(iii) $\leftrightarrow F = ma$		
	$R\sin\theta + \mu R\cos\theta = \frac{mv^2}{r}$	M1W1M1	
	$12\,740 \times \frac{5}{13} + 0.4 \times 12\,740 \times \frac{12}{13} = \frac{1000v^2}{50}$	W1	
	$v = 21.9\text{ms}^{-1}$	W1	12
8	**(i)** $F = ma$	M1	
	$-0.004v^2 - 0.2 \times 10 = 0.2a$	W2	
	$-0.02(500 + v^2) = \frac{v\text{d}v}{\text{d}x}$	MW1	
	(ii) $v\frac{\text{d}v}{\text{d}x} = -0.02(500 + v^2)$		
	$\int \frac{v}{500 + v^2}\text{d}v = \int -0.02\text{d}x$	M2W1	
	$\frac{1}{2}\ln\left\|500 + v^2\right\| = -0.02x + c$	W2	
	at $x = 0$ $v = 15$		
	$c = \frac{1}{2}\ln\left\|725\right\|$	MW1	
	$\frac{1}{2}\ln\left\|500 + v^2\right\| = -0.02x + \frac{1}{2}\ln\left\|725\right\|$		
	at maximum height $v = 0$	M1	
	$\frac{1}{2}\ln 500 - \frac{1}{2}\ln 725 = -0.02x$		
	$x = 9.29\text{m}$	W1	12
		Total	**75**