Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2009

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

AMF11

TUESDAY 13 JANUARY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that
$\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 The matrix **A** is given by

$$\begin{pmatrix} 10 & 6 \\ 3 & 3 \end{pmatrix}$$

(i) Show that the eigenvalues of **A** are 1 and 12 [5]

(ii) Find an eigenvector corresponding to the eigenvalue 1 [3]

2 **(i)** The matrix $\mathbf{N} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

What transformation does this matrix represent? [2]

The transformation represented by the matrix **M** maps the points P (2, –3) and Q (4, 0) onto P′ (–1, –5) and Q′ (4, –4) respectively.

(ii) Find the matrix **M**. [4]

The triangle OP′Q′ is the image of the triangle OPQ under the transformation represented by **M**.

(iii) Find the area of the triangle OP′Q′. [4]

The matrix **S** represents the combined effect of the transformation represented by **M** followed by the transformation represented by **N**.

(iv) Find the matrix **S**. [3]

3 Let the matrix $\mathbf{T} = \begin{pmatrix} 2 & 4 & 2 \\ \lambda & 12 & 5 \\ 1 & 8 & \lambda \end{pmatrix}$

(i) Find, in terms of λ, the determinant of **T**. [3]

(ii) Find the values of λ for which an inverse exists. [5]

(iii) If $\lambda = 2$, find the inverse of **T**. [6]

(iv) If $\lambda = 3$, find the value of μ for which the following system of equations has infinitely many solutions.

$$2x + 4y + 2z = \mu$$
$$\lambda x + 12y + 5z = 7$$
$$x + 8y + \lambda z = 6$$

[3]

4 **S** is the set of matrices $\begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix}$, where x is any real number.

(i) Prove that **S** is closed under matrix multiplication. [4]

(ii) Find the identity element of **S** under matrix multiplication. [2]

(iii) Find the inverse of $\begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix}$ under matrix multiplication. [2]

(iv) Assuming that matrix multiplication is associative, what can now be stated about the set **S** under matrix multiplication? [1]

5 Two circles have equations

$$x^2 + y^2 + 6y - 16 = 0$$

$$x^2 + y^2 - 24x - 12y + 80 = 0$$

Find the point of intersection of the circles and hence show that the circles touch externally.

[11]

6 **(a)** The complex numbers z_1 and z_2 are given by

$$z_1 = 10 + 5\text{i} \text{ and } z_2 = 2 - \text{i}$$

(i) Find $\frac{z_1}{z_2}$, giving your answer in the form $a + b\text{i}$, where a and b are real numbers. [5]

(ii) Find the modulus and argument of $\frac{z_1}{z_2}$ [4]

(b) **(i)** Sketch on an Argand diagram the locus of those points w which satisfy

$$|w - (10 + 5\text{i})| = |w - (2 - \text{i})|$$ [3]

(ii) On the same diagram sketch the locus of those points z which satisfy

$$|z - (10 + 5\text{i})| = 6$$ [3]

(iii) On your diagram shade the region which represents the locus of v where v satisfies both

$$|v - (10 + 5\text{i})| \leqslant |v - (2 - \text{i})| \text{ and } |v - (10 + 5\text{i})| \leqslant 6$$ [2]