Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2009

Mathematics

Assessment Unit C2

assessing

Module C2: AS Core Mathematics 2

[AMC21]

AMC21

FRIDAY 22 MAY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(a)** **(i)** Simplify $x(3x^2 + 2 + 4x^{-3})$ [1]

(ii) Hence, integrate with respect to x

$$x(3x^2 + 2 + 4x^{-3})$$ [4]

(b) Using the trapezium rule with 6 ordinates, find an approximate value for

$$\int_0^1 \frac{4}{(1+x^2)}\,dx$$ [6]

2 **(i)** A sequence is defined recursively by

$$u_{n+1} = \frac{2}{3}u_n \quad \text{where } u_1 = 1$$

Find u_2, u_3 and u_4 [3]

(ii) State whether this sequence is convergent or divergent. [1]

A geometric series is formed by adding the terms of the sequence to give

$$1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots.$$

(iii) Find the common ratio of this geometric series. [1]

(iv) Find the sum to infinity of this geometric series. [2]

3 In the binomial expansion of $(1 + nx)^{10}$, the coefficient of x^2 is 3 times the coefficient of x.
Find the value of n, where $n \neq 0$ [6]

4 **(i)** On the same diagram, sketch the curves $y = 2^x$ and $y = 1 + 2^x$.
Label any relevant points on the axes. [4]

The y coordinate of a point P on the curve $y = 1 + 2^x$ is 6

(ii) By solving the equation

$$1 + 2^x = 6$$

find the x coordinate of P.
[A solution by trial and improvement is not acceptable] [4]

5 **(a)** Prove the identity

$$\tan\theta + \frac{1}{\tan\theta} \equiv \frac{1}{\sin\theta\cos\theta}$$ [5]

(b) Solve the equation

$$\sin^2 x = \frac{1}{4}$$

for $-90° < x \leqslant 90°$ [4]

(c) Solve the equation

$$\cos 2x = 0.4$$

for $0 < x \leqslant \pi$ [4]

6 Shown in **Fig. 1** below is the curve $y = 4 + x^2$

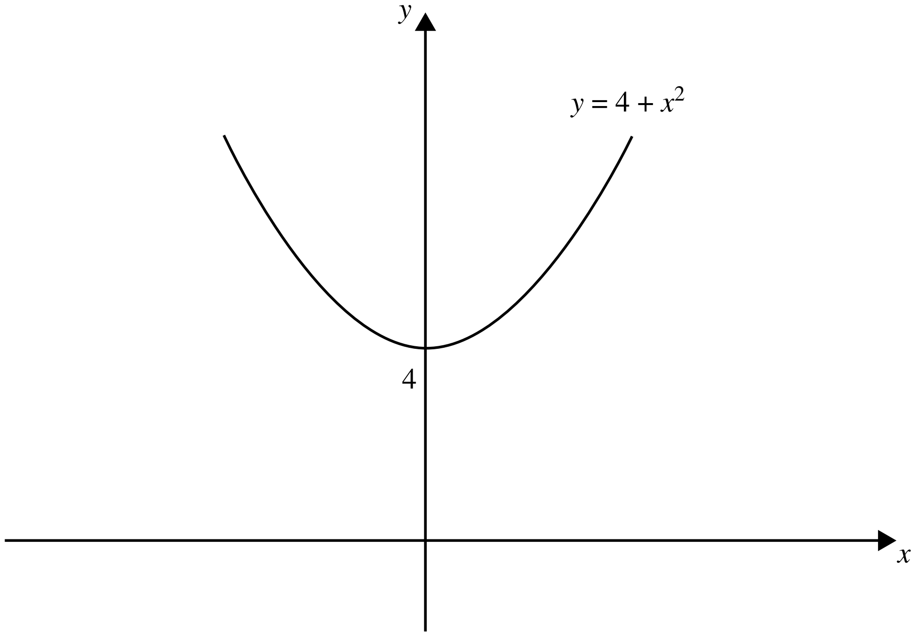

Fig. 1

(i) Find the area of the region bounded by the curve $y = 4 + x^2$, the x-axis, y-axis and the line $x = 1$ [5]

(ii) Hence, find the area of the region bounded by the curve $y = 4 + x^2$ and the line $y = 5$ [4]

7 The network coverage of a mobile phone mast M may be modelled as a circle as shown in **Fig. 2** below.

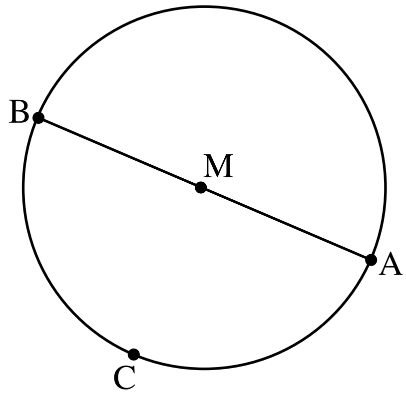

Fig. 2

Points A (2, 1), B(k, $k + 5$) and C (–1, –1) lie on the circumference of the circle, centre M. AB is a diameter of the circle.

(i) Find the gradient of AC. [2]

(ii) Hence, write down the gradient of BC and **prove** that $k = -3$ [4]

(iii) Find the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2$$ [5]

8 A silver medal is divided into two parts by a line AB.
The medal is in the shape of a circle, centre O, as shown in **Fig. 3** below.

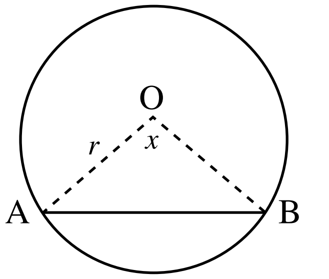

Fig. 3

The radius of the circle is r and the angle AOB is x radians.

(i) Write down the area of the minor sector OAB. [1]

(ii) Write down the area of the triangle AOB. [1]

The areas of the two parts of the medal divided by the line AB are in the ratio $5:1$

(iii) Show that

$$\sin x = x - \frac{\pi}{3}$$ [8]

THIS IS THE END OF THE QUESTION PAPER