Rewarding Learning

ADVANCED
General Certificate of Education
2009

Mathematics

Assessment Unit C3

assessing

Module C3: Core Mathematics 3

[AMC31]

AMC31

THURSDAY 28 MAY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Differentiate:

(i) $\dfrac{x}{4 - x^2}$ [4]

(ii) $(x^2 + 3)^5$ [3]

2 **(a)** Find the term in x^3 in the binomial expansion of

$$(1 + 2x)^{-1}$$ [4]

(b) Express $\dfrac{6x - 4}{(2x - 1)^2}$ in partial fractions. [6]

3 **(a)** A slide in an adventure playground can be modelled by the curve

$$y = 1 + 20e^{-x}$$

between $x = 1$ and $x = 10$ as shown in **Fig. 1** below.

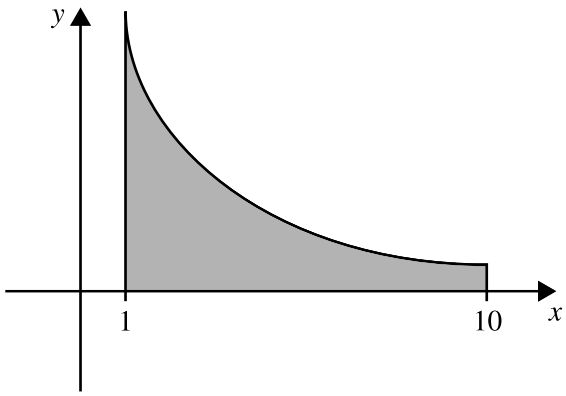

Fig. 1

Find the shaded area. [6]

(b) Find

$$\int \left(\frac{3}{x} - \frac{x}{5} + \sec 2x \tan 2x + 7 \right) dx$$ [5]

4 The graph of a function $y = f(x)$ is sketched below in **Fig. 2**.

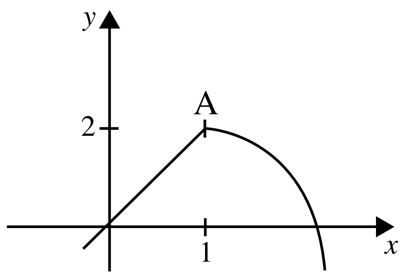

Fig. 2

On separate diagrams sketch the graphs of:

(i) $y = 3f(\frac{1}{2}x)$ [2]

(ii) $y = 4 - f(x)$ [2]

indicating the coordinates of the images of the point A.

5 **(i)** Show that the equation $2 - \ln x = x^2$ has a solution between $x = 1$ and $x = 2$ [4]

(ii) By taking $x = 1$ as a first approximation and using the Newton–Raphson method twice, find a better approximation to the solution of the equation $2 - \ln x = x^2$ [5]

6 A particle travels in a straight line in such a way that its distance x metres from a fixed point O at time t seconds can be given by the equation

$$x = 4 + \sqrt{3}\sin 2t + \cos 2t$$

(i) Find the initial distance of the particle from O. [1]

(ii) Find the rate of change of the distance of the particle from O at time t. [3]

(iii) Hence find the first time when the particle is at its greatest distance from O. [7]

7 **(a)** Solve the equation

$$\sec(2\theta - 30°) = -\frac{2}{\sqrt{3}}$$

for $-180° < \theta < 180°$ [7]

(b) Prove the identity

$$(\operatorname{cosec}^2\theta - 1)(\tan^2\theta + 1) \equiv \operatorname{cosec}^2\theta$$ [7]

8 Find the equation of the normal to the curve

$$y = x^2\ln(3x - 2) + 5$$

at the point on the curve where $x = 1$ [9]