ADVANCED
General Certificate of Education
2009

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

WEDNESDAY 20 MAY, AFTERNOON

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A bowl is formed by rotating through 2π radians about the x-axis, the arc of the curve

$$y = \sqrt{5x}$$

between $x = 0$ and $x = a$, where a is a positive constant.
The bowl is full of water.
Find the volume of water in the bowl. [6]

2 Two points A and B have coordinates $(1, 3, 4)$ and $(3, -2, 0)$ respectively.

(i) Find the distance between A and B. [2]

(ii) Find the vector equation of the line that passes through A and B. [5]

(iii) Show that the point $(5, -7, -4)$ lies on this line. [4]

3 Using the substitution $u = 1 + x$, find the **exact** value of

$$\int_{-1}^{0} x(1+x)^{\frac{1}{2}}\,dx$$

[8]

4 **(a)** Without using your calculator, find the **exact** value of $\tan 2A$ given that $\tan A = \frac{1}{7}$ and that A is acute. [3]

(b) Solve the equation

$$3 \cos \theta = \sin (\theta + 30°)$$

where $0° \leq \theta \leq 360°$ [7]

5 The functions f and g are defined as:

$$f : x \rightarrow 3x + 1 \qquad\qquad x \in \mathbb{R} \ x > 2$$

$$g : x \rightarrow \frac{1}{x} \qquad\qquad x \in \mathbb{R} \ x > 0$$

(i) State the range of f [1]

(ii) Find the composite function gf and state its domain and range. [5]

6 (i) Show that

$$\frac{d}{dx}\left(\frac{x}{1+x}\right) = \frac{1}{(1+x)^2} \qquad [4]$$

(ii) A curve has the equation

$$\frac{x}{1+x} - x^2 + \frac{y}{1+y} = 0$$

Find the gradient of the curve at the point (1,1) [6]

7 Given the differential equation

$$\frac{dy}{dx} = \frac{3y}{x+1}$$

and that $x = 1$ when $y = 16$, express y in terms of x [10]

8 Find

(i) $\displaystyle\int_0^2 xe^{-x}\,dx$ [7]

(ii) $\displaystyle\int \sin^3 x \ dx$ [7]

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