Rewarding Learning

ADVANCED
General Certificate of Education
2009

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

[AMF21]

AMF21

FRIDAY 19 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that
$\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Express

$$\frac{1}{(2x^2 + 3)(x - 1)}$$

in partial fractions. [5]

2 Find in radians the general solution of the equation

$$\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$$ [7]

3 Show that the sum of the series

$$1^3 + 3^3 + 5^3 + \ldots + (2n - 1)^3$$

is given by $n^2(2n^2 - 1)$. [7]

4 Given that one of the roots of

$$z^3 - z^2 + 3z + 5 = 0$$

is $z = 1 - 2\text{i}$, find the other 2 roots and plot all 3 roots on an Argand diagram. [6]

5 If

$$u_1 = 7 \text{ and } u_{n+1} = 3u_n - 2$$

prove by the method of mathematical induction that

$$u_n = 2(3^n) + 1, \text{ where } n \in \mathbb{Z}^+$$ [6]

6 Solve the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 36e^{-3x}$$

given that $y = 2$ and $\frac{dy}{dx} = 5$ when $x = 0$ [11]

7 **(i)** Using Maclaurin's theorem, derive a series expansion of $\sin\theta$ up to and including the term in θ^5 [5]

(ii) Using de Moivre's theorem, show that

$$\sin 3\theta \equiv 3\sin\theta - 4\sin^3\theta$$ [5]

(iii) Hence, find a series expansion for $\sin^3\theta$ up to and including the terms in θ^5 [4]

Please turn over for Question 8

8 The parabola $y^2 = 8x$ is shown in **Fig. 1** below.
F is the focus and P a point on the parabola. The normal to the parabola at P cuts the x-axis at G, and PP′ is a line parallel to the x-axis.

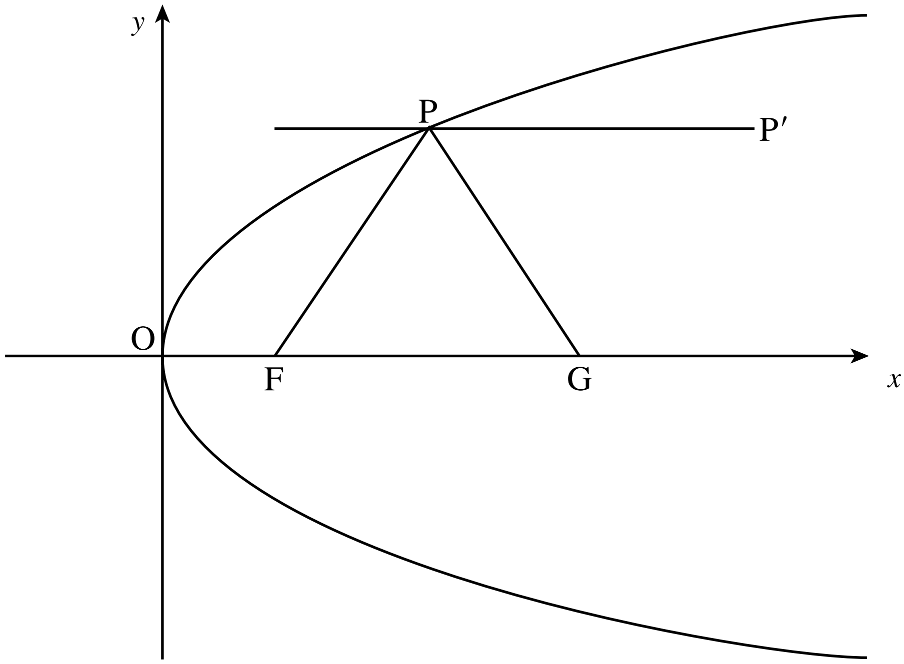

Fig. 1

(i) Write down the co-ordinates of F [1]

(ii) Verify that the point P is given parametrically by $(2t^2, 4t)$. [2]

(iii) Show that the equation of the normal PG is given by

$$y + tx = 2t^3 + 4t$$ [6]

(iv) Show that FP = FG [7]

(v) Prove that $F\hat{P}G = G\hat{P}P'$
This proves that light rays parallel to the axis of a parabolic mirror illuminate the focus.
[3]