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General Certificate of Education
2009

Mathematics

Assessment Unit M4

assessing

Module M4: Mechanics 4

[AMM41]

AMM41

WEDNESDAY 17 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that
$\ln z \equiv \log_e z$

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Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A guitar has six strings each of which produces a fundamental frequency. The frequency f depends on the tension P in the string, the length l of the string and the mass per unit length ρ of the string.
The frequency is related to the other variables as follows:

$$f = k P^{\alpha} l^{\beta} \rho^{\gamma}$$

where k is a dimensionless constant.

(i) Use the method of dimensions to find α and γ and show that $\beta = -1$ [7]

The fundamental frequency f_1 of the first string S_1 is four times that of the sixth string S_6
The mass per unit length of S_1 is ρ_1 and that of S_6 is ρ_6
Both strings have the same tension and length.

(ii) Show that $\rho_6 = 16\rho_1$ [3]

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2 Triangle ABC is isosceles with equal sides AB and AC.
Angle BAC is 2θ, where $\cos\theta = 0.8$
Forces of magnitude 10 N, 10 N and 30 N act along the sides AB, AC and BC respectively as shown in **Fig. 1** below.
The mid point of BC is M.

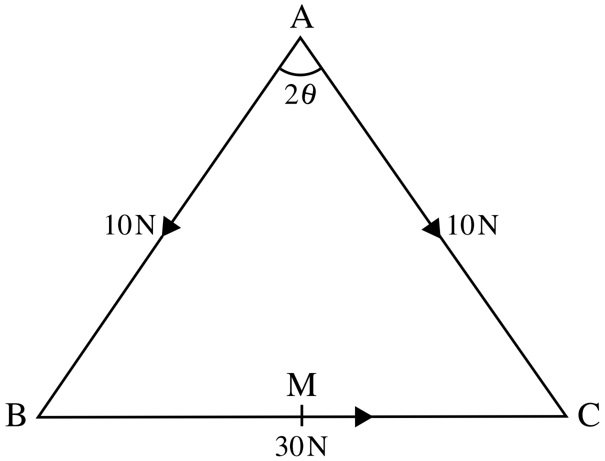

Fig. 1

(i) Find the resultant of the two forces of magnitude 10 N, stating its magnitude and two points on its line of action. [3]

(ii) Hence find the magnitude of the resultant of the three forces and identify one point on its line of action. [3]

The distance AB is 0.85 m.
The distance from B to the line of action of the resultant is d.

(iii) Find d. [5]

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3 **Take g = 10 m s^{-2} in this question.**

The Ratzan – a large rodent recently discovered on a tropical island – swings from tree to tree at the end of a long vine.

The weight of the Ratzan is 200 N.

A Ratzan was photographed at A moving at 16 m s^{-1} in a vertical circle holding the end of a 10 m vine that made an angle of 60° with the downward vertical.

The Ratzan, R, can be modelled as a particle on the end of a light inextensible rope whose other end is fixed at C, as shown in **Fig. 2** below.

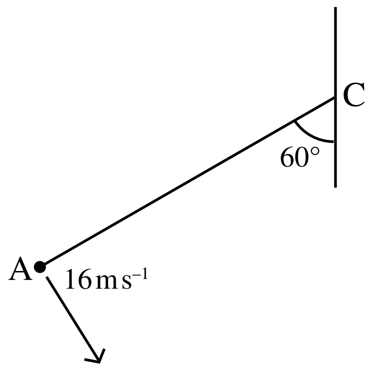

Fig. 2

(i) Find the tension in the vine when the Ratzan was at A. [5]

After leaving A the Ratzan swung the taut vine through 180° to B as shown in **Fig. 3** below.

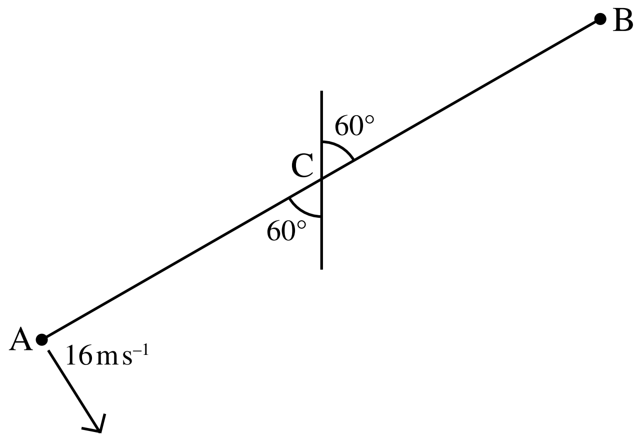

Fig. 3

(ii) Find the tension in the vine when the Ratzan was at B. [8]

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4 Two particles of masses $3m$ and $2m$ are moving towards one another along a straight horizontal track. Before they collide, their speeds are $3u$ and $2u$ respectively. Afterwards their velocities are v_1 and v_2 as shown in **Fig. 4** below.

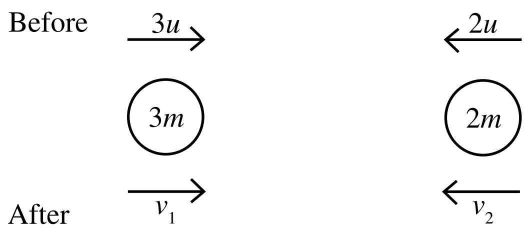

Fig. 4

The coefficient of restitution between the particles is e.

(i) Find v_1 and v_2 in terms of e and u. [7]

(ii) Show that the loss in kinetic energy caused by the collision is

$$15mu^2(1 - e^2)$$ [4]

(iii) If 37.5% of the initial kinetic energy is lost in the collision, find e. [3]

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5 A solid hemisphere has radius a, density ρ and volume $V = \dfrac{2\pi a^3}{3}$

(i) Write down an expression for its mass in terms of a and ρ. [1]

A solid hemisphere is produced by rotating the area bounded by the curve $y = \sqrt{(a^2 - x^2)}$, between $x = 0$ and $x = a$, and the x-axis through 2π radians about the x-axis as shown in **Fig. 5** below.

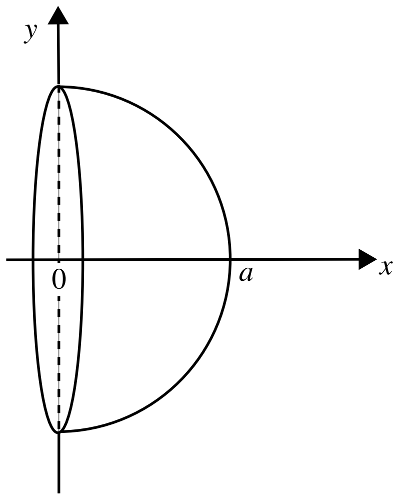

Fig. 5

(ii) Find, in terms of a, the x coordinate of the centre of mass of the hemisphere. [7]

An ornamental bowl is made by removing a solid hemisphere of radius 15 cm from a larger one of radius 20 cm.

(iii) Find the distance of the centre of mass of the bowl below the rim of the bowl. [6]

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6 At a certain instant the artificial satellite Xeo, X, lies directly between the Earth, centre E, and the Moon, centre M. The distance EM is d metres.
The resultant of the gravitational pulls of the Earth and Moon on Xeo is zero at the point X where EX = fd and $0 < f < 1$, as shown in **Fig. 6** below.

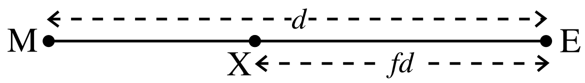

Fig. 6

The masses of the Earth and Moon are M_E and M_M respectively.
The universal gravitational constant is G.

(i) Show that

$$\frac{M_E}{M_M} = \frac{f^2}{(1-f)^2}$$ [5]

(ii) Given that $M_E = 81M_M$, find the value of f. [3]

Some time later the Sun, centre S, Moon, Xeo and Earth are in line and in that order. The distance EX = 0.75ME.
The masses of the Sun, Moon and Earth are 1.99×10^{30} kg, 7.35×10^{22} kg and 5.98×10^{24} kg respectively.
The distance SX is 1.49×10^{11} m and ME is 3.84×10^{8} m.
The mass of Xeo is 500 kg.
The value of G is 6.67×10^{-11} m^3 kg^{-1} s^{-2}

(iii) Find the resultant of the gravitational forces of the Sun, Moon and Earth acting on Xeo. [4]

With the configuration of Sun, Moon and Earth as above and the Sun's gravitational field included, the position of zero resultant gravity between the Earth and Moon would be closer to the Earth.

(iv) How do the results of **(ii)** and **(iii)** show this? [1]

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