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ADVANCED
General Certificate of Education
January 2010

Mathematics

Assessment Unit C3

assessing

Module C3: Core Mathematics 3

[AMC31]

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FRIDAY 15 JANUARY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that
$\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(a)** Simplify as far as possible

$$\frac{x^2+x-12}{x^2-16} \div \frac{x-3}{x^2-4x}$$ [5]

(b) Express

$$\frac{3x+3}{(x-1)(x+2)}$$

in partial fractions. [6]

2 Use Simpson's rule with 5 ordinates to find an approximation for

$$\int_0^4 \frac{1}{1+x^3}\,\mathrm{d}x$$ [6]

3 **(a)** Find the binomial expansion of

$$\sqrt{1-x^2}$$

up to and including the term in x^4 [6]

(b) Find the exact values of x for which

$$|\ln x| = 3$$ [5]

4 **Fig. 1** below shows a sketch of the graph of the function $y = \mathrm{f}(x)$.

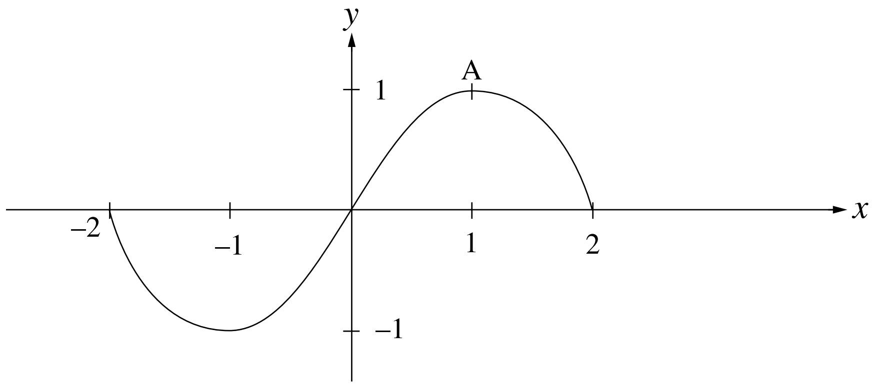

Fig. 1

On separate diagrams sketch the graphs of:

(i) $y = -\mathrm{f}(x + 2)$; [2]

(ii) $y = 3\mathrm{f}(\frac{1}{2}x)$ [2]

marking clearly the image of the point A on each sketch.

5 **(a)** Differentiate

(i) $(3x^2 - 4)^6$ [3]

(ii) $\dfrac{\ln x}{x^2 - 1}$ [4]

(b) Find

$$\int \left(\frac{5}{x} - \cos 2x + \operatorname{cosec}^2 x - 2x \right) \mathrm{d}x$$ [5]

6 The temperature, $H°$ centigrade, of the heating element in an electric heater, t seconds after it has been switched off, is given by

$$H = 10 + 60e^{-kt}$$

where k is a constant.

(i) Find the initial temperature of the element. [2]

The heating element takes 30 seconds to reach 20 °C.

(ii) Show that $k = 0.0597$ to 3 significant figures. [4]

(iii) Find the rate at which the temperature of the element is changing after 1 minute. [4]

7 **(a)** Prove the identity:

$$\operatorname{cosec}^2\theta + \sec^2\theta \equiv \operatorname{cosec}^2\theta \sec^2\theta \qquad [6]$$

(b) Find the **exact** values of x given that

$$3\tan^2 x - 5\sec x + 1 = 0$$

where $-\pi < x \leqslant \pi$ [7]

8 **Fig. 2** below shows a drawing of a capstan.

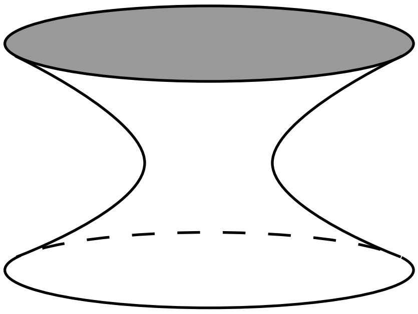

Fig. 2

Fig. 3 below shows the cross-section through a vertical plane containing the centre of the capstan.

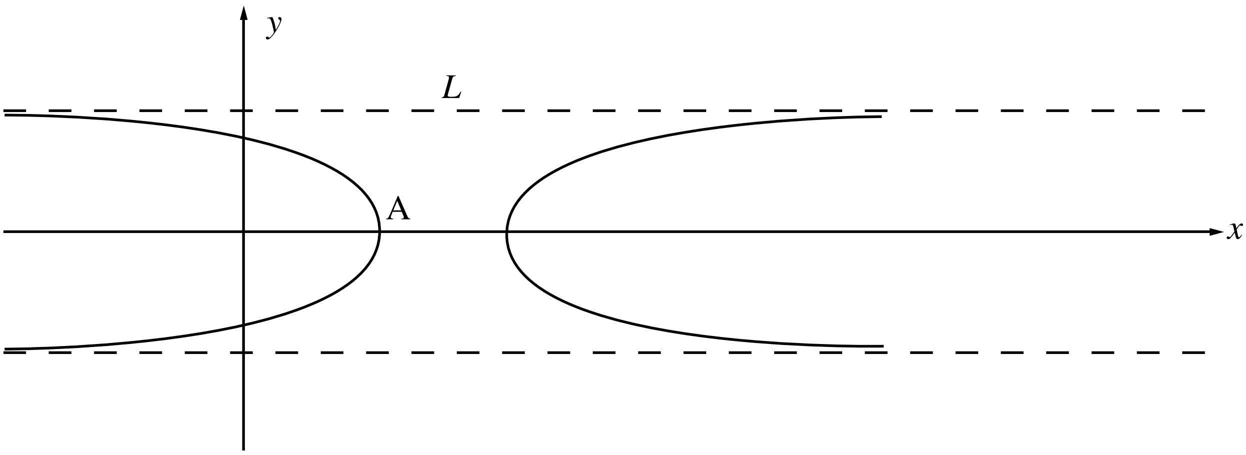

Fig. 3

The outline of the cross-section can be modelled by the parametric equations

$$x = 3 - \frac{1}{\sin\theta}, \qquad y = 2\cos\theta$$

(i) Find a corresponding Cartesian equation. [4]

(ii) Hence or otherwise, find the coordinates of the point A at which the curve crosses the x-axis and write down the equation of the asymptote L. [4]

THIS IS THE END OF THE QUESTION PAPER