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ADVANCED
General Certificate of Education
January 2010

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

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FRIDAY 29 JANUARY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Relative to a fixed origin O,
point A has position vector $2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and
point C has position vector $-4\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$

(i) Find a vector equation of the line AC. [4]

The points OABC are the vertices of a parallelogram.

(ii) Find the position vector $\overrightarrow{OB}$. [2]

(iii) Hence find the acute angle between the diagonals OB and AC. [5]

2 Let

$$g(x) = \begin{cases} x^3 & 0 \leqslant x \leqslant 2 \\ 4x & 2 \leqslant x \leqslant 5 \end{cases}$$

and

$$h(x) = \begin{cases} x^3 & 0 \leqslant x \leqslant 2 \\ 4x + 1 & 2 \leqslant x \leqslant 5 \end{cases}$$

(a) Which of g or h is a function? Give a reason for your answer. [2]

(b) A function f is defined as

$$f(x) = 4 - x^2 \qquad x \in \mathbb{R}$$

(i) Sketch the graph of $y = f(x)$. [2]

(ii) Hence state the range of $f(x)$. [1]

(iii) Write down two functions $a(x)$ and $b(x)$ such that $f(x)$ is equal to the composite function $ab(x)$.
State the domains of the two functions. [3]

3 **(i)** Rewrite $(8 \sin \theta + 6 \cos \theta)$ in the form

$$R \sin (\theta + \alpha)$$

where R is an integer and $0 \leqslant \alpha \leqslant \frac{\pi}{2}$ [3]

(ii) Hence state the maximum and minimum values of

$$8 \sin \theta + 6 \cos \theta$$ [2]

(iii) A mass is suspended from the end of a spring, as shown in **Fig. 1** below.

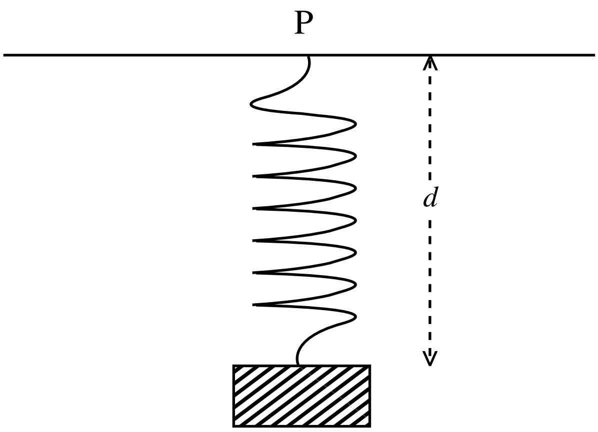

Fig. 1

The mass is oscillating.
After t seconds the distance d (cm) between the fixed point P and the mass is given by

$$d = 15 + 8 \sin 2t + 6 \cos 2t$$

Find the time at which the mass is first at its lowest point. [4]

4 **(i)** Differentiate

$$x^3 - 3x^2y + 2y^2 = 3$$

implicitly with respect to x. [5]

(ii) Hence find the equation of the tangent to the curve

$$x^3 - 3x^2y + 2y^2 = 3$$

at the point (1, 2). [3]

5 Solve the differential equation

$$\left(\sin^2\theta\right)\frac{\mathrm{d}x}{\mathrm{d}\theta} = \frac{4}{x^2}$$

given that $x = 3$ when $\theta = \frac{\pi}{4}$ [7]

6 A trophy is to be made in the shape of a rugby ball.
It can be modelled by the volume generated when the area between the curve

$$y = \sin x$$

and the x-axis, between $x = 0$ and $x = \pi$, is rotated through 2π radians about the x-axis, as shown in **Fig. 2** below.

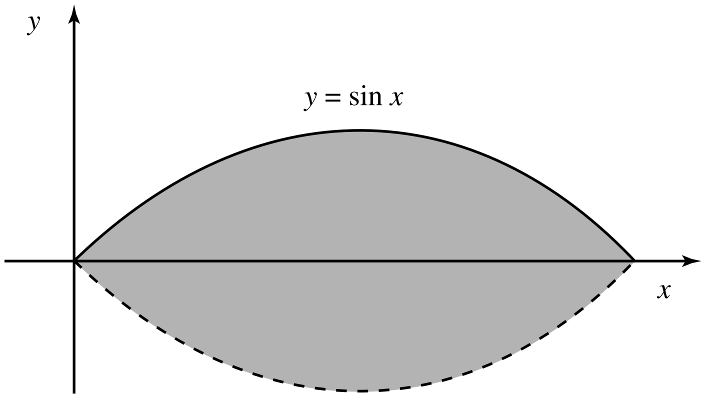

Fig. 2

Find the **exact** volume of the trophy. [9]

7 **(a)** Sketch the graph of

$$y = \cot x \qquad \text{for } -180° \leqslant x \leqslant 180°$$ [2]

(b) Prove the identity

$$\frac{1}{\sin 2\theta} + \cot 2\theta \equiv \cot \theta$$ [7]

8 **(a)** Find $\int 2x^4 \ln 3x \, dx$ [6]

(b) Use partial fractions to find

$$\int \frac{x + 9}{3 - 2x - x^2} \, dx$$ [8]

THIS IS THE END OF THE QUESTION PAPER