GCE AS

Mathematics

January 2010

Mark Schemes

Issued: April 2010

**NORTHERN IRELAND GENERAL CERTIFICATE OF SECONDARY EDUCATION (GCSE)
AND NORTHERN IRELAND GENERAL CERTIFICATE OF EDUCATION (GCE)**

MARK SCHEMES (2010)

Foreword

Introduction

Mark Schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of 16- and 18-year-old students in schools and colleges. The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes therefore are regarded as a part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

The Council hopes that the mark schemes will be viewed and used in a constructive way as a further support to the teaching and learning processes.

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General Certificate of Education
January 2010

Mathematics

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assessing

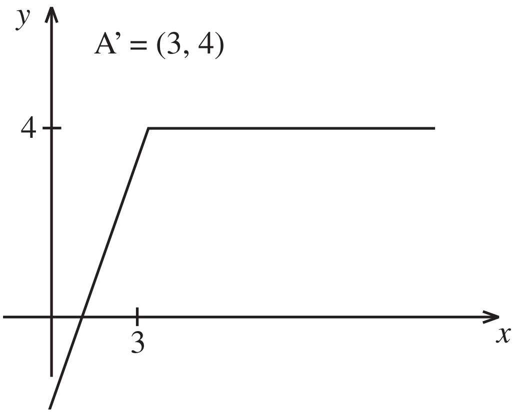
Module C1: AS Core Mathematics 1

[AMC11]

MONDAY 11 JANUARY, MORNING

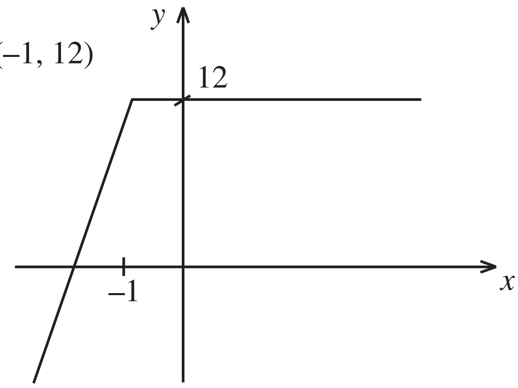
MARK SCHEME

		AVAILABLE MARKS
1 **(i)**		



M1W1

(ii)



M1W1 — 4

2

$60x + 20y + 10z = 1800$
$100x + 30y + 5z = 2500$
$80x + 40y + 15z = 2600$ — M1W2

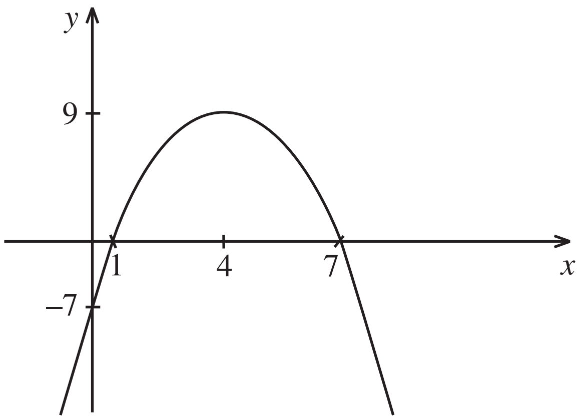
$200x + 60y + 10z = 5000$
$60x + 20y + 10z = 1800$
$140x + 40y = 3200$ — M1W1

$300x + 90y + 15z = 7500$
$80x + 40y + 15z = 2600$
$220x + 50y = 4900$ — MW1

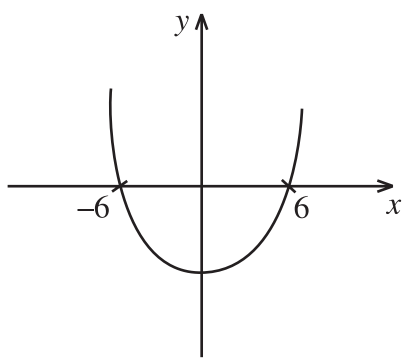
$880x + 200y = 19\,600$
$700x + 200y = 16\,000$
$180x = 3600$

$x = 20$ — MW1
$y = 10$ — W1
$z = 40$ — W1 — 9

				AVAILABLE MARKS
3	**(i)**	$m_{AB} = \frac{y-5}{-2-1} = \frac{y-5}{-3}$	M1W1	
	(ii)	$m_{BC} = \frac{y+3}{-4}$	MW1	
		$\frac{y-5}{-3} = \frac{4}{y+3}$ or $\frac{y-5}{-3} \times \frac{y+3}{-4} = -1$	M1	
		$(y-5)(y+3) = -12$	MW1	
		$y^2 - 2y - 3 = 0$		
		$(y-3)(y+1) = 0$	M1	
		$y = 3$ or $y = -1$	W1	7
4	**(a)**	$[6x^2 + 9x - 2x - 3 - 8x + 2] \div \frac{3x+1}{3x-1}$	M1W1	
		$[6x^2 - x - 1] \div \frac{3x+1}{3x-1}$	MW1	
		$[6x^2 - x - 1] \times \frac{3x-1}{3x+1}$	M1	
		$[(3x+1)(2x-1)] \times \frac{3x-1}{3x+1}$	MW1	
		$(2x-1)(3x-1)$	W1	
	(b)	$\frac{20+4\sqrt{5}}{3+\sqrt{5}}$	M1	
		$\frac{20+4\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$	M1W1	
		$\frac{40-8\sqrt{5}}{4} = 10 - 2\sqrt{5}$	MW2	
	(c)	$\frac{2^{4x}}{2^{(x-1)}} = 2^{\frac{1}{2}}$	M1W1	
		$2^{3x+1} = 2^{\frac{1}{2}}$	M1W1	
		$3x + 1 = \frac{1}{2}$	M1	
		$x = \frac{-1}{6}$	W1	17

			AVAILABLE MARKS
5 **(i)**	$x^2 - 8x + 7 = [(x-4)^2 - 16 + 7]$	M1W1	
	$= (x-4)^2 - 9$	MW1	
(i)	Alternative solution		
	$x^2 + 2px + p^2 + q = x^2 - 8x + 7$	M1	
	$p = -4$	MW1	
	$q = -9$	W1	
(ii)	Min value = –9	MW1	
	When $x = 4$	MW1	
(iii)	$x^2 - 8x + 7 = 0$		
	$(x-7)(x-1)$	M1	
	$x = 7$ or $x = 1$	W2	
(iv)		M1W1	10
6 **(i)**	$f(2) = 200 - 200 - 8 + 8 = 0$	M1W1	
(ii)	$$\begin{array}{r} 25x^2 \qquad\quad -4 \\ x-2\overline{)25x^3 - 50x^2 - 4x + 8} \\ \underline{25x^3 - 50x^2} \qquad\qquad \\ 0 - 4x + 8 \\ \underline{-4x + 8} \\ 0 \end{array}$$	M2W1	
	$(x-2)(5x-2)(5x+2)$	MW1	
(iii)	$(x-2)(5x-2)(5x+2) = 0$		
	$x = 2$ or $x = \frac{2}{5}$ or $x = \frac{-2}{5}$	MW3	9

			AVAILABLE MARKS
7 **(a)** **(i)** $\frac{dy}{dx} = 6x^2 - 8x$	MW3		
(ii) $x = 2 \quad m = 24 - 16 = 8$	MW1		
$x = 2 \quad y = 16 - 16 - 3 = -3$	MW1		
$y + 3 = 8(x - 2)$	M1		
$y = 8x - 19$	W1		
(b) $P = 16t^{\frac{1}{2}} + 27t^{-1}$			
$\frac{dP}{dt} = 8t^{\frac{-1}{2}} - 27t^{-2}$	M1W2		
$\frac{dP}{dt} = 8t^{\frac{-1}{2}} - 27t^{-2} > 0$	M1		
$\frac{8}{\sqrt{t}} > \frac{27}{t^2}$			
$t^{\frac{3}{2}} > \frac{27}{8}$			
$t > \frac{9}{4}$	W1		12
$\frac{9}{4} < t < 10$			
8 $3x^2 - 2 = mx - 5$	M1		
$3x^2 - mx + 3 = 0$	MW1		
$b^2 - 4ac < 0$	M2		
$m^2 - 36 < 0$	MW1		



$-6 < m < 6$	MW2	7
	Total	**75**

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2010

Mathematics

Assessment Unit C2

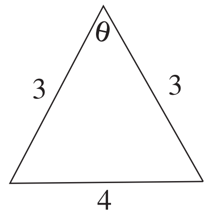
assessing

Module C2: AS Core Mathematics 2

[AMC21]

MONDAY 25 JANUARY, MORNING

MARK SCHEME

		AVAILABLE MARKS
1 (i) 		
$\sin\frac{\theta}{2} = \frac{2}{3}$	M1W1	
$\frac{\theta}{2} = 0.7297$ rad		
$\theta = 1.46$ rad	W1	
or Cosine Rule $4^2 = 3^2 + 3^2 - 2 \times 3 \times 3 \times \cos\theta$	M1W1	
$\theta = 1.46$ rad	W1	
(ii) Area Sector $= \frac{1}{2}r^2\theta$	M1	
$= \frac{1}{2} \times 9 \times 1.46$		
$= 6.57$ unit^2	W1	
(iii) Area triangle $= \frac{1}{2}ab\sin c$	M1	
$= \frac{1}{2} \times 3 \times 3 \times \sin 1.46$		
$= 4.472$ unit^2	W1	
or $\frac{1}{2}$ base × ht $= \frac{1}{2} \times 4 \times \sqrt{3^2 - 2^2} = 4.472$		
$\therefore$ area segment $= 6.57 - 4.472 = 2.10$ unit^2	M1W1	
(iv) $(x - a)^2 + (y - b)^2 = r^2$	M1	
$(x - 2)^2 + (y - 1)^2 = 9$	W2	12

			AVAILABLE MARKS
2	**(i)** $\dfrac{(x^2+2)^2}{x^2} = \dfrac{x^4+4x^2+4}{x^2}$	M1W1	
	$= x^2 + 4 + \dfrac{4}{x^2} \therefore B = C = 4$	MW1	
	(ii) $\displaystyle\int_1^2 \frac{(x^2+2)^2}{x^2}\,\mathrm{d}x = \int_1^2 x^2 + 4 + \frac{4}{x^2}\,\mathrm{d}x$		
	$= \left[\dfrac{x^3}{3} + 4x - \dfrac{4}{x}\right]_1^2$	MW3	
	$= \left(\dfrac{8}{3} + 8 - 2\right) - \left(\dfrac{1}{3} + 4 - 4\right)$	M1	
	$= 8\tfrac{1}{3}$	W1	8
3	**(i)** Let $S_n = a + [a+d] + [a+2d] + \ldots\ldots + [a+(n-1)d]$	MW1	
	then $S_n = [a+(n-1)d] + [a+(n-2)d] + \ldots\ldots + a$	M1	
	add $2S_n = [2a+(n-1)d] + [2a+(n-1)d] + \ldots\ldots + [2a+(n-1)d]$	M1	
	$= n[2a+(n-1)d]$	W1	
	$\therefore S_n = \dfrac{n}{2}[2a+(n-1)d]$	MW1	
	(ii) $a + a + d = 2$		
	$2a + d = 2$	M1W1	
	$a + 40d = 475$	M1W1	
	$2a + d = 2$		
	$\underline{2a + 80d = 950}$	M1	
	$79d = 948$		
	$d = 12$	W1	
	$a = -5$	MW1	
	(iii) $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$		
	$S_{20} = 10\,[-10 + 19 \times 12]$		
	$= 2180$	MW2	14

			AVAILABLE MARKS
4	$\frac{10 \times 9 \times 8}{1 \times 2 \times 3} 2^7(-x)^3$	MW3	
	$= -15360x^3$	W1	4

5 **(i)** Area $\approx \frac{h}{2}$ [1st + last + 2 × others] — M1

$h = 10$ — MW1

Area $= \frac{10}{2}[0 + 1.31 + 2(1.16 + 2.48 + 5.25 + 3.79 + 6.24)]$ — W1

$= 195.75$

$= 196\,\text{m}^2$ — W1

(ii) Area $= \int_0^{60} \frac{1}{180}\left(x^2 - 60x\right)\text{d}x$ — M2W1

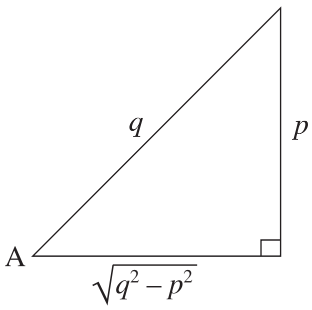
$= \frac{1}{180}\left[\frac{x^3}{3} - 30x^2\right]_0^{60}$ — MW2

$= -200$

$= 200\text{m}^2$ — W1

(iii) The equation for y does not at all model the deep trough around $x = 50$ or $x = 60$ when $y = 0$ — MW1 — 11

6 **(a)**



M1W1

$\tan A = \frac{p}{\sqrt{q^2 - p^2}}$ — MW1

$\tan^2 A = \frac{p^2}{q^2 - p^2}$ — MW1

		AVAILABLE MARKS
(b) $\frac{1}{2}\tan x - \sin x = 0$		
$\frac{1}{2}\frac{\sin x}{\cos x} - \sin x = 0$	MW1	
$\sin x\left(\frac{1}{2\cos x} - 1\right) = 0$	MW1	
$\sin x = 0$ or $\cos x = \frac{1}{2}$	MW2	
$x = 0°, \pm 180°, \pm 60°$	MW3	11
7 **(a)** $\log_5 15 + 2\log_5 2 - \log_{25} 9$		
	M1W1	
$\log_{25} 9 = \frac{\log_5 9}{\log_5 25}$	W1	
$\log_5 15 + \log_5 4 - \log_5 3$ $\quad = \frac{1}{2}\log_5 9$	M1W1	
$\log_5 \frac{15 \times 4}{3}$	M2	
$\log_5 20$	W1	
(b) **(i)** $P(1 + 0.05)^t$	MW2	
(ii)	MW1	
$P(1+0.05)^t = \frac{3}{2}P$		
$(1+0.05)^t = \frac{3}{2}$	M1	
$\log(1+0.05)^t = \log\frac{3}{2}$	M1W1	
$t\log(1+0.05) = \log\frac{3}{2}$		
$t = \frac{\log 1.5}{\log 1.05}$	W1	
$= 8.31$ years		15
(9)	**Total**	**75**

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2010

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

WEDNESDAY 20 JANUARY, AFTERNOON

MARK SCHEME

AVAILABLE MARKS

1 $x^2 + y^2 - 2x - 24 = 0$ ①

$x^2 + y^2 - 6x - 8y + 20 = 0$ ②

Subtract to give:

$4x + 8y - 44 = 0$ M1

$\Rightarrow x = -2y + 11$ W1

Substitute into equation ①

$\Rightarrow (-2y + 11)^2 + y^2 - 2(-2y + 11) - 24 = 0$ M1W1

$\Rightarrow 4y^2 - 44y + 121 + y^2 + 4y - 22 - 24 = 0$

$\Rightarrow 5y^2 - 40y + 75 = 0$ MW1

$\Rightarrow y^2 - 8y + 15 = 0$

$\Rightarrow (y - 3)(y - 5) = 0$

$\Rightarrow y = 3, 5$ W2

Hence $x = -6 + 11 = 5$, $x = -10 + 11 = 1$ W1

This gives the points (5, 3) and (1, 5) 8

AVAILABLE MARKS

2 **(a)** A rotation of 90° clockwise about the origin O. MW3

(b) **(i)** det **Q** = 6 – 5 = 1 MW1

(ii) When **Q** is used as a transformation matrix it will leave the area of a shape unchanged by the transformation. MW1

(iii) **R** = **QP** M1M1

$$= \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & 3 \\ -2 & 1 \end{pmatrix}$$ MW1

(iv) $$\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -5 & 3 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$ M1

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-5+6} \begin{pmatrix} 1 & -3 \\ 2 & -5 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$ M1

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ 2 & -5 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$ W1

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$$ MW1

Hence A has coordinates (4, 7)

Alternative Solution

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -5 & 3 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$ M1

Hence $1 = -5x + 3y$ ① W1

Hence $-1 = -2x + y$ ②

Hence Equation ① – 3 × Equation ② gives

$4 = x$ MW1

Using ② gives $y = 7$ MW1

Hence A has coordinates (4, 7) 12

AVAILABLE MARKS

3 **(i)** $\det = \begin{vmatrix} 2 & 1 & a+1 \\ 3 & a & 2 \\ -1 & -3 & 3 \end{vmatrix}$

$= 2\begin{vmatrix} a & 2 \\ -3 & 3 \end{vmatrix} - 1\begin{vmatrix} 3 & 2 \\ -1 & 3 \end{vmatrix} + (a+1)\begin{vmatrix} 3 & a \\ -1 & -3 \end{vmatrix}$ M1

$= 2(3a + 6) - 1(9 + 2) + (a + 1)(-9 + a)$ W1

$= 6a + 12 - 11 - 9a - 9 + a^2 + a$

$= a^2 - 2a - 8$ MW1

(ii) If $a = 3 \Rightarrow \det = 9 - 6 - 8 \neq 0$ M1W1

Hence there is one unique solution MW1

(iii) $a = 4 \Rightarrow \det = 16 - 8 - 8 = 0$ MW1

Hence there is either no solution or an infinite number of solutions.

$2x + y + 5z = 4$ ① M1

$3x + 4y + 2z = 2$ ②

$-x - 3y + 3z = 6$ ③

① – ② gives $-x - 3y + 3z = 2$ M1

which is inconsistent with equation ③.

Hence there are no solutions. MW1 10

		AVAILABLE MARKS
4 **(i)** Zero rotation = Identity = I	MW1	
Rotation of 72° clockwise about centre = P		
Rotation of 144° clockwise about centre = Q	MW1	
Rotation of 216° clockwise about centre = R		
Rotation of 288° clockwise about centre = S	MW1	

(ii)

	I	P	Q	R	S
I	I	P	Q	R	S
P	P	Q	R	S	I
Q	Q	R	S	I	P
R	R	S	I	P	Q
S	S	I	P	Q	R

MW5

(iii)

	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

MW3

(iv) G and H are isomorphic since they are both cyclic groups of order 5 MW1 12

5 **(i)** $|\mathbf{M} - \lambda\mathbf{I}| = 0$ M1

$$\begin{vmatrix} 4-\lambda & -2 & 0 \\ -2 & 8-\lambda & 1 \\ 0 & 1 & 4-\lambda \end{vmatrix} = 0$$ W1

$\Rightarrow (4-\lambda)[(8-\lambda)(4-\lambda) - 1] + 2[-2(4-\lambda)] = 0$ M1W1

$\Rightarrow (4-\lambda)[32 - 12\lambda + \lambda^2 - 1 - 4] = 0$

$\Rightarrow (4-\lambda)[\lambda^2 - 12\lambda + 27] = 0$ W1

$\Rightarrow (4-\lambda)(\lambda - 9)(\lambda - 3) = 0$ W1

$\Rightarrow \lambda = 4, 3, 9$ W1

(ii) If $\lambda = 3$ then

$$\begin{pmatrix} 4 & -2 & 0 \\ -2 & 8 & 1 \\ 0 & 1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$ M1

$\Rightarrow 4x - 2y = 3x$ $\quad x = 2y$ M1

$\Rightarrow -2x + 8y + z = 3y$

$\Rightarrow y + 4z = 3z$ $\quad y = -z$ W1

Check with 2nd equation $\Rightarrow -4y + 8y - y = 3y$

$\Rightarrow 3y = 3y$

Hence an eigenvector is $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ W1

AVAILABLE MARKS

(iii) $\begin{pmatrix} 4 & -2 & 0 \\ -2 & 8 & 1 \\ 0 & 1 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 8 \end{pmatrix}$ M1W1

Since this equals $4\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ we find that $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ is an eigenvector MW1

Also $\begin{pmatrix} 4 & -2 & 0 \\ -2 & 8 & 1 \\ 0 & 1 & 4 \end{pmatrix}\begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} -18 \\ 45 \\ 9 \end{pmatrix} = 9\begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix}$ MW1

Hence $\begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix}$ is an eigenvector

(iv) $\mathbf{P} = \begin{pmatrix} 2 & 1 & -2 \\ 1 & 0 & 5 \\ -1 & 2 & 1 \end{pmatrix}$ M1W1 17

AVAILABLE MARKS

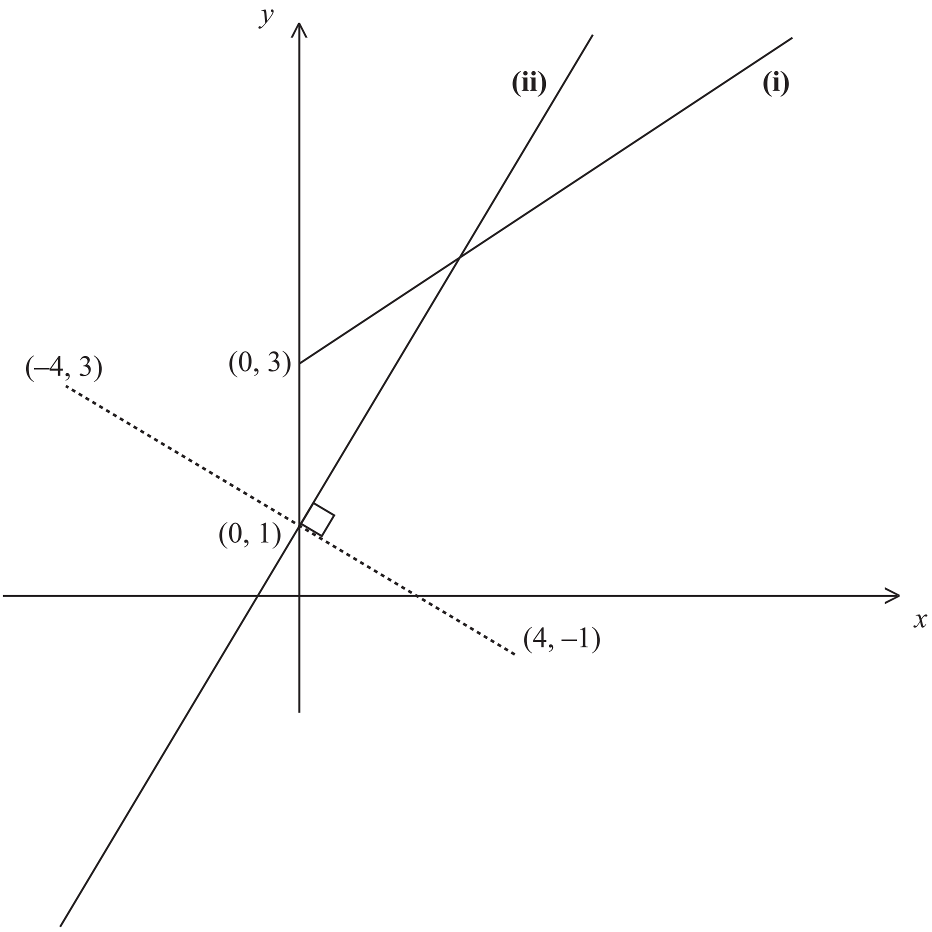
6 **(a)** $\frac{1}{p} = \frac{1}{3+2i} \times \frac{3-2i}{3-2i}$ M1

$= \frac{3-2i}{9+4}$ W1

$= \frac{3}{13} - \frac{2}{13}i$ W1

(b) **(i)** Straight half line through the point (0, 3) with gradient 1 MW3

(ii) Perpendicular bisector of the line joining (4, –1) and (–4, 3) MW3



AVAILABLE MARKS

(iii) First line has equation $y = x + 3$	MW1	
Second line: Gradient of line joining (4, –1) and (–4, 3) is given by $$\frac{3+1}{-4-4} = -\frac{1}{2}$$	M1	
Hence gradient of perpendicular is 2	MW1	
Line passes through midpoint which is (0, 1)	W1	
Hence equation is $y = 2x + 1$	MW1	
Alternative solution for second line		
$\lvert w - 4 + i \rvert = \lvert w + 4 - 3i \rvert$		
When $w = x + yi$		
then $(x - 4)^2 + (y + 1)^2 = (x + 4)^2 + (y - 3)^2$	M1W1	
$\Rightarrow x^2 - 8x + 16 + y^2 + 2y + 1 = x^2 + 8x + 16 + y^2 - 6y + 9$	W1	
$\Rightarrow 8y = 16x + 8$		
Hence equation is $y = 2x + 1$	MW1	
To find point of intersection $x + 3 = 2x + 1$	M1	
Hence $x = 2$ which gives $y = 5$	W1	
Hence point of intersection is (2, 5)		16
	Total	**75**

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2010

Mathematics

Assessment Unit M1

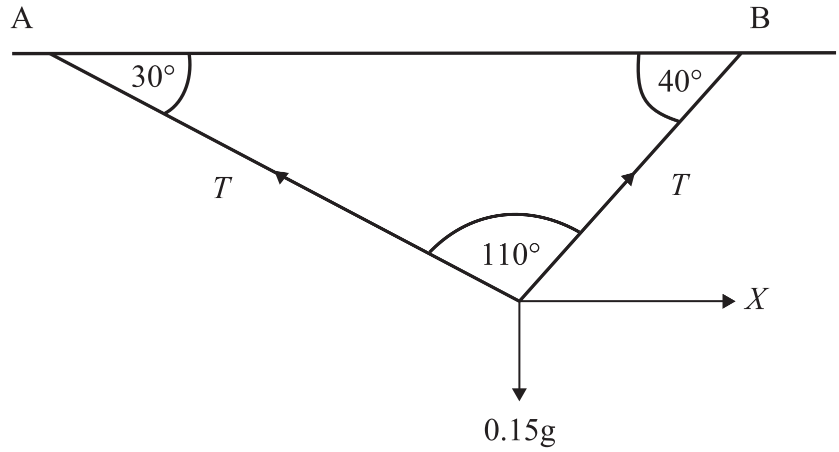
assessing

Module M1: Mechanics 1

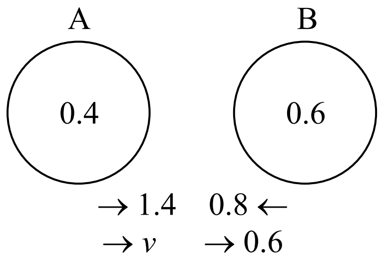
[AMM11]

WEDNESDAY 20 JANUARY, AFTERNOON

MARK SCHEME

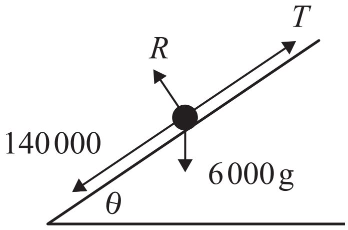
				AVAILABLE MARKS
1	$F = ma$	M1		
	$a = 0$	M1		
	$T - Fr = 0$	W1		
	$Fr = \mu R$	M1		
	$= 0.2 \times 3\text{g}$	W1		
	$T = 0.2 \times 3\text{g}$			
	$= 5.88\,\text{N}$	W1		6
2	**(i)** 	MW2		
	(ii) $\updownarrow$ $0.15\text{g} = T\sin 30° + T\sin 40°$	M1W1		
		M1		
	$T = 1.29\,\text{N}$	W1		
	$\longleftrightarrow$ $T\cos 30° = X + T\cos 40°$	M1W1		
	$X = 0.129\,\text{N}$	W1		9

3 $\rightarrow +$



		AVAILABLE MARKS
momentum before = momentum after	M1	
$0.4 \times 1.4 - 0.6 \times 0.8 = 0.4\,v + 0.6 \times 0.6$	M1W2	
$0.56 - 0.48 = 0.4\,v + 0.36$		
$v = -0.7$	MW1	
speed $0.7\,\text{m}\,\text{s}^{-1}$ direction of motion reversed	MW1	6

4 **(i)**



(diagram)	MW2	
(ii) $F = ma$	M1	
$T - 140\,000 - 6\,000\,\text{g} \sin\theta = -6\,000 \times 2$	MW3	
$T - 140\,000 - 36\,000 = -12\,000$		
$T = 164\,000\,\text{N}$	W1	7

AVAILABLE MARKS

5 (i) $v = 3t^2 - 4t$

$t = 1$ $\quad v = 3 - 4 = -1\ \text{m s}^{-1}$ MW1

(ii) $s = \int 3t^2 - 4t\, dt$ M1

$= t^3 - 2t^2 + c$ W1

at $t = 0$ $\quad s = 3 \quad \therefore c = 3$ M1W1

$s = t^3 - 2t^2 + 3$

(iii) $v = 3t^2 - 4t$

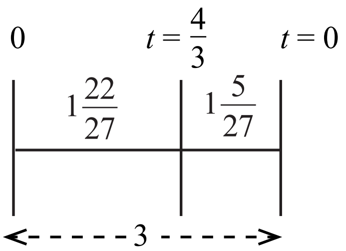
$v = 0$ M1

stops at $t = 0$ or $\frac{4}{3}$ W1

at $t = \frac{4}{3}$ $\quad s = \left(\frac{4}{3}\right)^3 - 2\left(\frac{4}{3}\right)^2 + 3$ MW1

$s = 1\frac{22}{27}$

$t = \frac{4}{3}$ [1] $t = 0$

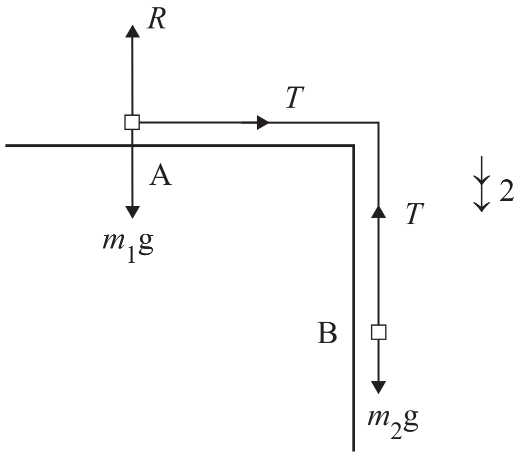


M1

MW1

$\therefore$ total distance is $2\frac{10}{27}$ m MW1 11

6 (i)



MW2

(ii) A $\quad T = 2m_1$ M1W1

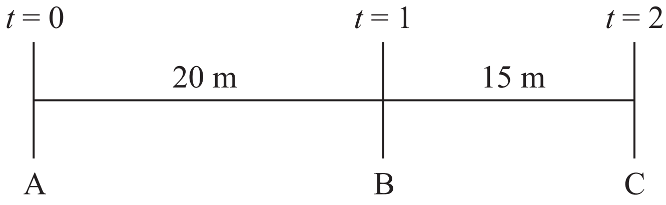
B $\quad m_2 g - T = 2m_2$

$T = 8m_2$ MW1

$\therefore 2m_1 = 8m_2$ M1

$\frac{m_1}{m_2} = \frac{8}{2} = 4$ W1 7

				AVAILABLE MARKS
7 (i)				

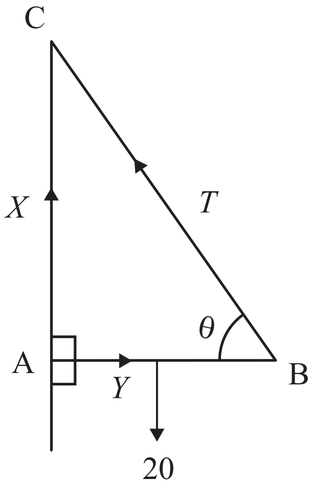


A → B	$u = u$	$S = ut + \frac{1}{2}at^2$	M1	
	$t = 1$			
	$s = 20$	$20 = u + \frac{1}{2}a$	W1	
	$a = a$			
A → C	$u = u$	$S = ut + \frac{1}{2}at^2$	M1	
	$t = 2$			
	$s = 35$	$35 = 2u + 2a$	W1	
	$a = a$		MW1	
		$35 = 2u + 2a$	M1	
		$40 = 2u + a$		
		$-5\,\text{m s}^{-2} = a$	W1	
		deceleration $5\,\text{m s}^{-2}$		
		$20 = u - \frac{1}{2} \times 5$		
		$u = 22.5\,\text{m s}^{-1}$	MW1	

(ii)	$u = 22.5$			
	$v = 0$	$v = u + at$	M1	
	$a = -5$	$0 = 22.5 - 5t$	MW1	
	$t = ?$	$t = 4.5\,\text{s}$	W1	
		further time $2\frac{1}{2}$ seconds	MW1	12

		AVAILABLE MARKS

8 **(i)**



MW2

(ii) $T \sin \theta \times 2 = 20 \times 1$ — M1

$T = 11.5\,\text{N}$ — M1W2

W1

(iii) $\updownarrow$ $X + \dfrac{40}{\sqrt{12}} \times \sin \theta = 20$ — M1

$X = 10.0\,\text{N}$ — W1

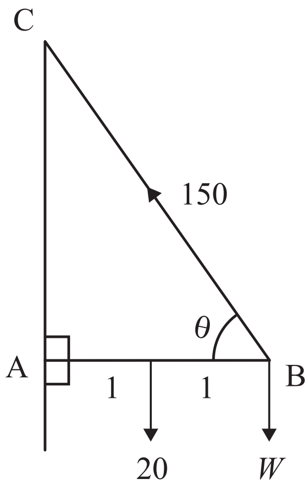
$\longleftrightarrow$ $Y = T \cos \theta$ — M1

$= 5.77\,\text{N}$ — W1

$\text{mag}^n \text{ resultant} = \sqrt{10^2 + 5.77^2}$ — M1

$= 11.5\,\text{N}$ — W1

(iv)



A) $20 \times 1 + W \times 2 = 150 \sin \theta \times 2$ — M1W2

$W = 120\,\text{N}$ — W1 — 17

Total — **75**

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2010

Mathematics

Assessment Unit S1

assessing

Module S1: Statistics 1

[AMS11]

WEDNESDAY 27 JANUARY, AFTERNOON

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

					AVAILABLE MARKS
1	(i)	Advantage:	More manageable amount of data	M1	
		Disadvantage:	Possibility unrepresentative	M1	
	(ii)	Advantage:	More concise than using raw data or frequency table	M1	
		Disadvantage:	Loss of actual values may lead to inaccuracies	M1	
	(iii)	Midvalues:	2.5 7.5 12.5 17.5 22.5	MW1	
		From calculator $n = 56$ $\sum fx = 455$ $\sum fx^2 = 5200$		M1	
		$\bar{x} = 8.125$ m			
		$= 8.13$ (3sf)		W1	
		$\sigma_{n-1} = 5.2277\ldots$m		M1	
		$\sigma^2_{n-1} = 27.329\ldots\text{m}^2$ (27.3m^2)		W1	9
2	(i)	$0.16 + k + 0.25 + k + 0.31 = 1$		M1	
		$k = 0.14$		W1	
	(ii)	$E(X) = (-2) \times 0.16 + (-1) \times 0.14 + 0 \times 0.25 + 1 \times 0.14 + 2 \times 3.1$		M1	
		$= 0.3$		W1	
		$E(X^2) = (-2)^2 \times 0.16 + (-1)^2 \times 0.14 + 0^2 \times 0.25 + 1^2 \times 0.14 + 2^2 \times 3.1$		M1	
		$= 2.16$		W1	
		$\text{Var}(X) = E(X^2) - [E(X)]^2 = 2.16 - 0.3^2 = 2.07$		M1 W1	
	(iii)	$E(Y) = 1 - 4\,(E(X)) = 1 - 4 \times 0.3 = -0.2$		MW1	
		$\text{Var}\,(X) = (-4)^2\ \text{Var}\,(X)$		M1	
		$= 16 \times 2.07 = 33.12$			
		$= 33.1$ (3sf)		W1	11

			AVAILABLE MARKS
3 **(i)**	Let X be r.v. "No. of vehicles during a one-minute period"		
	then $X \sim \text{Po}(8)$		
	$P(X = 6) = \dfrac{e^{-8}\ 8^6}{6!} = 0.122$ (3sf)	M1 W1	
(ii)	Let Y be r.v. "No. of vehicles during a 15 second period"		
	then $Y \sim \text{Po}(2)$	MW1	
	$P(Y \geq 2) = 1 - P(Y < 2)$	M1	
	$= 1 - P(Y = 0 \text{ or } 1)$		
	$= 1 - \left[\dfrac{e^{-2}\ 2^0}{0!} + \dfrac{e^{-2}\ 2^1}{1!}\right]$	W2	
	$= 1 - 3e^{-2}$		
	$= 0.59399 \ldots = 0.594$ (3sf)	W1	
(iii)	Passing singly / Random occurance / Independent events	M1	8
4 **(i)**	Let X be r.v. "No. of sixes scored"		
	then $X \sim B(8, 0.25)$	M1	
	$P(X = 3) = \binom{8}{3}(0.25)^3(0.75)^5$	M1	
	$= 0.208$ (3sf)	W1	
(ii)	$P(X \geq 3) = 1 - P(X > 3)$	M1	
	$= 1 - P(X = 0, 1, 2)$		
	$= 1 - \left[\binom{8}{0}(0.25)^0(0.75)^8 + \binom{8}{1}(0.25)^1(0.75)^7 + \binom{8}{2}(0.25)^2(0.75)^6\right]$	MW3	
	$= 1 - [0.1001 + 0.26696 + 0.31146]$		
	$= 0.321$ (3sf)	W1	
(iii)	$P(X = 5 \mid X \geq 3) = \dfrac{P(X = 5 \cap X \geq 3)}{P(X \geq 3)} = \dfrac{P(X = 5)}{P(X \geq 3)}$	M1W1	
	$P(X = 5) = \binom{8}{5}(0.25)^5(0.75)^3 = 0.0231$	MW1	
	$P(X = 5 \mid X \geq 3) = \dfrac{0.0231}{0.321} = 0.0718$ (3sf)	W1	12

AVAILABLE MARKS

5 Let X be r.v. "the mass, in grams, of bags of potatoes"

$X \sim N(\mu, 40^2)$

(i) $P(X > 2678.4) = 0.025$

$P(X < 2678.4) = 0.975$ MW1

$P\left(Z < \frac{2678.4 - \mu}{40}\right) = 0.975$ MW1

$\frac{2678.4 - \mu}{40} = \Phi^{-1}(0.975) = 1.96$ M1 W1

$\mu = 2678.4 - 40 \times 1.96$

$= 2600$ W1

(ii) $P(2540 < X < 2610) = P\left(\frac{2540 - 2600}{40} < Z < \frac{2610 - 2600}{40}\right)$

$= P(-1.5 < Z < 0.25)$ W2

$= \Phi(0.25) - \Phi(-1.5)$

$= \Phi(0.25) - (1 - \Phi(1.5))$

$= \Phi(0.25) + \Phi(1.5) - 1$ M1

$= 0.5987 + 0.9332 - 1$ W2

$= 0.5319$

$= 0.532$ (3sf) W1 11

AVAILABLE MARKS

6 **(i)** $E(X) = \int_0^6 \frac{1}{108} x(6x^2 - x^3)\,dx = \frac{1}{108}\int_0^6 (6x^3 - x^4)\,dx$ — M2

$= \frac{1}{108}\left[\frac{3x^4}{2} - \frac{x^5}{5}\right]_0^6 = \frac{388.8}{108} = 3.6$ — W1 W1

(ii) $P(X < 2) = \int_0^2 \frac{1}{108}(6x^2 - x^3)\,dx$ — M1

$= \frac{1}{108}\left[2x^3 - \frac{x^4}{4}\right]_0^2 = \frac{1}{108}(16 - 4) = \frac{1}{9}$ — W1, W1

(iii) $P(2 < X < 4) = \int_2^4 \frac{1}{108}(6x^2 - x^3)\,dx$ — M1

$= \frac{1}{108}\left[2x^3 - \frac{x^4}{4}\right]_2^4$

$= \frac{1}{108}\left[2(4)^3 - \frac{4^4}{4}\right] - \frac{1}{9} = \frac{16}{27} - \frac{1}{9} = \frac{13}{27}$ (0.481 3sf) — W1

(iv) $P(X > 4) = 1 - \left(\frac{1}{9} + \frac{13}{27}\right) = \frac{11}{27}$ — M1 W1

E(charge) = $\sum$ [(charge) × P(charge)] — M1

$= 2.5 \times \frac{1}{9} + 3.5 \times \frac{13}{27} + 4.5 \times \frac{11}{27}$ — W1

= £3.80 — W1 — 14

			AVAILABLE MARKS
7	M – Milk chocolate P – Plain chocolate		
(i)	P(MP or PM) = $\frac{3}{6} \times \frac{3}{5} + \frac{3}{6} \times \frac{3}{5}$	M1 W1	
	$= \frac{3}{10} + \frac{3}{10} = \frac{3}{5}$	W1	
(ii)	P(Transfer MM) = $\frac{3}{6} \times \frac{2}{5} = \frac{1}{5}$	MW1	
	P(Transfer PP) = $\frac{3}{6} \times \frac{2}{5} = \frac{1}{5}$	MW1	
	P(Transfer MM then choose P) = $\frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$	M1 MW1	
	P(Transfer MP or PM then choose P) = $\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$	MW1	
	P(Transfer PP then choose P) = $\frac{1}{5} \times \frac{3}{5} = \frac{3}{25}$	MW1	
	P(Choose P) = $\frac{1}{25} + \frac{6}{25} + \frac{3}{25} = \frac{10}{25} = \frac{2}{5}$	W1	10
		Total	**75**