Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2010

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

AMF11

THURSDAY 24 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that
$\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or a scientific calculator in this paper.

1 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 2 & 7 \\ 3 & -2 \end{pmatrix}$$

(i) Show that the eigenvalues of **A** are ± 5 [5]

(ii) Find a unit eigenvector corresponding to the eigenvalue 5 [4]

2 A system of linear equations is given by

$$9x + 2y + az = 0$$

$$x - y - 3z = 0$$

$$(a - 1)x + y + 3z = 0$$

(i) Find the values of a for which there is a solution other than $x = y = z = 0$ [6]

(ii) For the value $a = 6$ find the general solution. [5]

3 The permutations

$$p = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \qquad q = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \qquad r = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

$$s = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \qquad t = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \qquad u = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

form a group under composition.

(i) Copy and complete the following group table.

Note that q ∘ r = s indicates that r followed by q produces s.

∘	p	q	r	s	t	u
p	p	q	r	s	t	u
q	q	p	s	r	u	t
r	r	t	p	u	q	s
s	s	u	q			
t	t	r	u			
u	u	s	t			

[6]

(ii) Find the period of the element s. [2]

(iii) Write down the inverse of the element r. [2]

(iv) Write down a subgroup of order 3 [2]

4 The equation of a circle is given by

$$x^2 + y^2 + 4x - 10y + 9 = 0$$

(i) Find the equation of the tangent to the circle at the point (2, 7). [5]

(ii) Show that this tangent passes through the point (3, 5). [1]

(iii) Hence, or otherwise, find the equation of the other tangent to the circle from the point (3, 5). [5]

5 **(a)** **(i)** Write down the matrix **M** which represents a rotation of 180° about the origin O. [2]

(ii) Find the image of the point (7, –3) under the transformation represented by **M**. [2]

(b) The matrix $\mathbf{N} = \begin{pmatrix} 1 & 2 \\ 4 & -7 \end{pmatrix}$

(i) The transformation represented by **N** maps a region S, of area $3\,\text{cm}^2$, to a new region T.

Find the area of T. [3]

(ii) Under the transformation represented by **N**, the line $y = mx$ is reflected in the x-axis.

Find the possible values of m. [8]

6 The complex numbers z_1 and z_2 are given by

$$z_1 = \sqrt{2} + \sqrt{2}\,\mathrm{i} \text{ and } z_2 = 1 + \sqrt{3}\,\mathrm{i}$$

(i) Find the modulus and argument of each of z_1 and z_2 [6]

(ii) On a clearly labelled Argand diagram, plot the points representing z_1, z_2 and $z_1 + z_2$ [5]

(iii) Hence, without using a calculator, show that

$$\tan\frac{7\pi}{24} = \frac{\sqrt{2}+\sqrt{3}}{1+\sqrt{2}}$$ [6]