Rewarding Learning

ADVANCED
General Certificate of Education
2010

Mathematics

Assessment Unit M4

assessing

Module M4: Mechanics 4

[AMM41]

AMM41

FRIDAY 18 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\ \text{m s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that
$\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A lamina, density ρ kg m^{-2}, is bounded by the x- and y-axes and the curve

$$y = (4 - x)^{\frac{1}{2}}$$

The curve crosses the axes at A (0, 2) and B (4, 0) as shown in **Fig. 1** below.

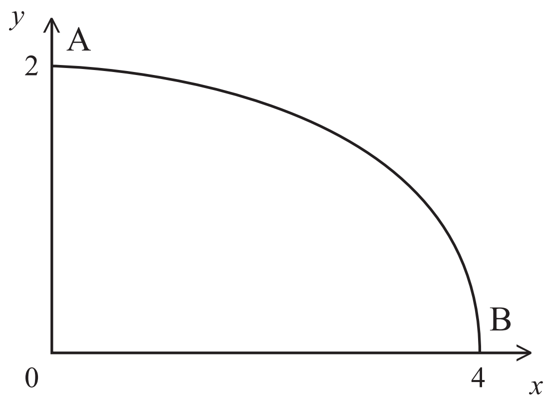

Fig. 1

(i) Show that the mass of the lamina is $\dfrac{16\rho}{3}$ kg. [5]

(ii) Show that the moment of the mass of the lamina about the x-axis is

$$\frac{\rho}{2}\int_0^4 (4 - x)\,\mathrm{d}x \ \text{ kg m}$$ [3]

The centre of mass of the lamina is at G.

(iii) Find the distance of G from the x-axis. [3]

2 The framework ABCDE of six light pin-jointed rods is freely hinged to a rigid vertical wall at A and B. It carries a load of 30 newtons at D as shown in **Fig. 2** below.

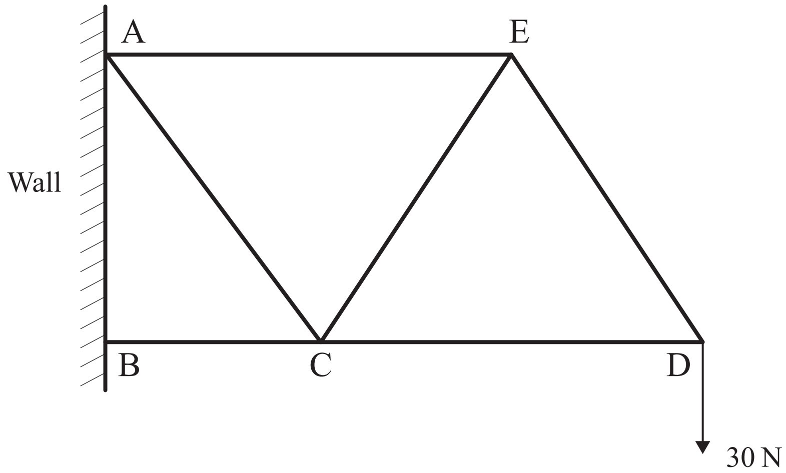

Fig. 2

AE, BC and CD are horizontal.
AE = CD = 1.6 m, BC = 0.8 m.
AC = CE = ED = 1.7 m.

(i) By considering the forces acting at D, explain briefly why the rod DE is in a state of tension and rod CD is in a state of thrust. [2]

(ii) Hence find these forces in DE and CD. [4]

(iii) Explain briefly why the reaction of the wall at B is a horizontal force. [1]

(iv) Find the magnitude of the reaction of the wall at A. [5]

3 Investigators have arrived at the scene of an accident. Witnesses have reported that the lorry involved simultaneously skidded and toppled outwards.

(i) Explain briefly why the normal reaction of the road on the inner set of wheels was zero at that instant. [1]

The lorry had been moving at v m s^{-1} in a horizontal circle of radius r metres around a bend banked at α to the horizontal.
The centre of mass G of the loaded lorry is d metres from either side of the lorry and h metres above the road surface as shown in **Fig. 3** below.

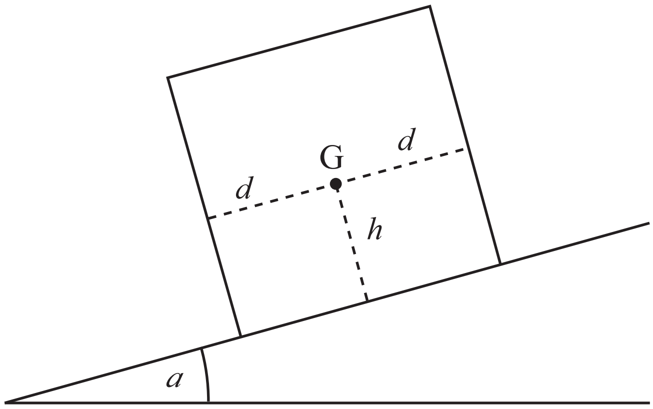

Fig. 3

The coefficient of friction between the sets of wheels and the road is μ.

(ii) Find μ in terms of d and h. [5]

The lorry had a high load and $h = 1.25d$.

(iii) Find the value of μ. [1]

The investigators know that

$$v^2 = \frac{rg(\tan\alpha + \mu)}{1 - \mu\tan\alpha}$$

and that $\tan\alpha = \frac{7}{66}$ and $r = 20\,\text{m}$.

(iv) Find the maximum speed at which the lorry could have negotiated the bend. [3]

4 A particle P of mass 1 kg is moving in a straight line at 20 m s^{-1} when it directly impacts on a particle Q of mass m which is at rest. P rebounds at v m s^{-1} and Q moves forward at $2v$ m s^{-1} in P's original direction of motion.

(i) Find m in terms of v. [3]

Half of the kinetic energy lost by P is transferred to Q.

(ii) Find v. [6]

The coefficient of restitution between the particles is e.

(iii) Find e. [3]

5 A small ring of mass m is threaded on to a fixed smooth vertical circular hoop ABP centre C and radius r. The diameter AB is vertical and the angle ACP is θ as shown in **Fig. 4** below.

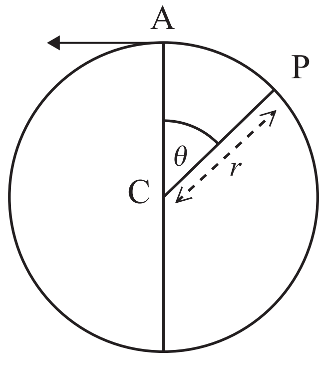

Fig. 4

The ring is projected horizontally from A with speed $\sqrt{2gr}$
The ring moves round the hoop through B eventually passing through A again.

The reaction between the hoop and the ring at P is R.

(i) Show that the magnitude of R is $mg(4 - 3\cos\theta)$. [10]

(ii) Explain briefly why R always points towards C. [2]

(iii) Show that $R_{max} = 7R_{min}$ [2]

6 In her laboratory Irina is investigating the period of oscillation P of torsion pendulums.
P is believed to be related to the moment of inertia of the disc I, the modulus of rigidity of the wire n, its length l and radius r as follows:

$$P = kn^s I^t l^u r^v$$

where k is a dimensionless constant and the indices have to be determined.

The dimensions of n are $[ML^{-1}T^{-2}]$ and those of I are $[ML^2]$

Use the method of dimensions to:

(i) find s and t; [6]

(ii) find v in terms of u. [2]

(iii) Show that a possible formula for P is

$$P = k\sqrt{\frac{I}{nr^3}}\left(\frac{l}{r}\right)^u$$ [2]

Irina was using copper wire of a particular radius and finding how the period varied as she changed only the length of the wire.
She discovered that when l was 0.4 m, P was 1.7725 s and when l was 1.6 m, P was 3.545 s.

(iv) Using the formula in **(iii)** find u. [4]

For the disc $I = 2.2 \times 10^{-5}$ kg m^2
For the copper wire $n = 4.4 \times 10^{10}$ kg m^{-1} s^{-2} and $r = 2 \times 10^{-4}$ m.

(v) Find k. [2]

THIS IS THE END OF THE QUESTION PAPER