GCE AS

Mathematics

Summer 2010

Mark Schemes

Issued: October 2010

**NORTHERN IRELAND GENERAL CERTIFICATE OF SECONDARY EDUCATION (GCSE)
AND NORTHERN IRELAND GENERAL CERTIFICATE OF EDUCATION (GCE)**

MARK SCHEMES (2010)

Foreword

Introduction

Mark Schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of 16 and 18-year-old students in schools and colleges. The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes therefore are regarded as a part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

The Council hopes that the mark schemes will be viewed and used in a constructive way as a further support to the teaching and learning processes.

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[AMC11]

WEDNESDAY 9 JUNE, AFTERNOON

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

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When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

1 (i) Midpoint P = (1, –2)   MW1

(ii) Grad $m = \dfrac{-7-3}{4--2} = \dfrac{-10}{6} = \dfrac{-5}{3}$ M1W1

Perp. grad $= \dfrac{3}{5}$ MW1

$y + 2 = \dfrac{3}{5}(x-1)$ M1W1

$3x - 5y - 13 = 0$ 6

2 (i) f(–1) = –a – 3 – b + 6 = 12   M2W1
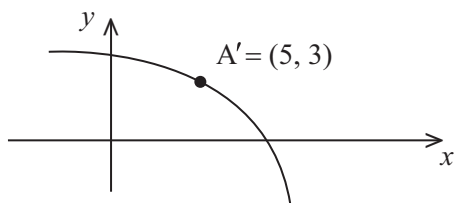a + b = –9

(ii) f(3) = 27a – 27 + 3b + 6 = 0   M1W1
9a + b = 7

(iii) 9a + b = 7
a + b = –9
8a = 16
a = 2   MW1
b = –11   MW1   7

3 (i)

A′ = (5, 3)   M1W1

(ii)

A′ = (5, –4)   M1W1

(iii)

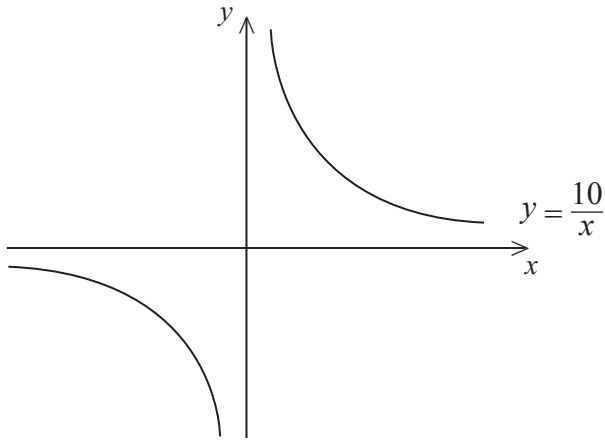A′ = (2.5, 4)   M1W1   6
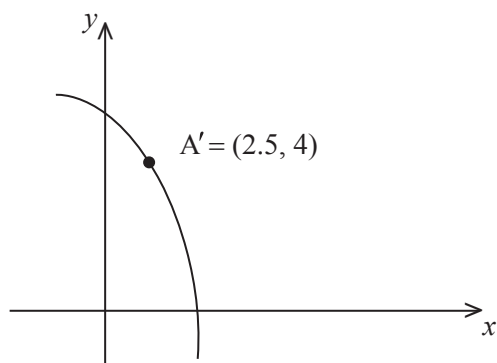
4 **(a)** $\dfrac{(3x-2)(3x+2)}{2x+1} \div \dfrac{3x-2}{3(2x+1)}$ M1W1

$\dfrac{(3x-2)(3x+2)}{2x+1} \times \dfrac{3(2x+1)}{3x-2}$ MW1

$3(3x+2)$ M1W1

(b) $\dfrac{(3-\sqrt{7}) \times (\sqrt{7}+2)}{(\sqrt{7}-2) \times (\sqrt{7}+2)}$ M1W1

$\dfrac{3\sqrt{7}+6-7-2\sqrt{7}}{7-4} = \dfrac{\sqrt{7}-1}{3}$ MW2

(c) $3^{x+1} = \dfrac{3^{3x}}{3^2}$ M1W1

$3^{x+1} = 3^{3x-2}$ MW1

$x+1 = 3x-2$ M1

$x = \dfrac{3}{2}$ W1 14

5 **(i)** $\dfrac{dy}{dx} = 4x^3 - 6x^2$ MW2

(ii) $\dfrac{dy}{dx} = 4x^3 - 6x^2 = 0$ M1

$x^2(4x-6) = 0$

$x = 0 \text{ or } x = \dfrac{3}{2}$ W2

$\dfrac{d^2y}{dx^2} = 12x^2 - 12x$ MW1

$x = \dfrac{3}{2} \Rightarrow \dfrac{d^2y}{dx^2} = 27 - 18 \Rightarrow +\text{ve} \therefore \min \text{ at } x = \dfrac{3}{2}$ MW1

$x = -1 \quad \dfrac{dy}{dx} = -4 - 6 = -10$ M1

$x = +1 \quad \dfrac{dy}{dx} = 4 - 6 = -2 \therefore \text{ Point of inflection at } x = 0$ W1 9

6 (i)

M1
MW1

$y = \dfrac{10}{x}$

(ii)

$y = 3x + 13$

MW1

$y = \dfrac{10}{x}$

13

$\dfrac{-13}{3}$

(iii) $3x + 13 = \dfrac{10}{x}$ M1

$3x^2 + 13x - 10 = 0$ W1

$(3x - 2)(x + 5) = 0$ M1

$x = \dfrac{2}{3}$ or $x = -5$ W2

$y = 15$ or $y = -2$ W1   9

7 (a) (i) $2x(x + 10) = 800$ MW1
$2x^2 + 20x - 800 = 0$ MW1
$x^2 + 10x - 400 = 0$

(ii) $x = \dfrac{-10 \pm \sqrt{100 + 1600}}{2}$ M1

$x = \dfrac{-10 \pm \sqrt{1700}}{2}$ W1

Width $= 2x = -10 + 10\sqrt{17}$ MW1

Length $= x + 10 = 5 + 5\sqrt{17}$

    MW1

(b) $b^2 - 4ac > 0$

$9 - 4p(5 - p) > 0$

$4p^2 - 20p + 9 > 0$ M2

$(2p - 1)(2p - 9)$ W1

$p = \dfrac{1}{2}$ or $p = \dfrac{9}{2}$ M1

$p < \dfrac{1}{2}$ or $p > 4\dfrac{1}{2}$ W2

    MW1   13

8 (i) $P = 2l + 2\pi r$ MW1

(ii) $400 = 2l + 2\pi r$ M1
$l = 200 - \pi r$ W1

(iii) $A = 2lr$ MW1
$A = 2(200 - \pi r)r$ MW1
$A = 400r - 2\pi r^2$

$\dfrac{dA}{dr} = 400 - 4\pi r$ M1W1

$400 - 4\pi r = 0$ M1

$r = \dfrac{100}{\pi}$ W1

$\dfrac{d^2A}{dr^2} = -4\pi \Rightarrow -\text{ve} \qquad \therefore \text{ max}$ MW1

$l = 100$ m   MW1   11

Total   **75**

Rewarding Learning

ADVANCED
General Certificate of Education
2010

Mathematics

Assessment Unit C2

assessing

Module C2: Core Mathematics 2

[AMC21]

THURSDAY 27 MAY, MORNING

MARK
SCHEME

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1 **(i)** $u_{n+1} = \dfrac{2}{1+u_n}$

$u_1 = 3$

$u_2 = \dfrac{1}{2}$

$u_3 = \dfrac{4}{3}$

$u_4 = \dfrac{6}{7}$

$u_5 = \dfrac{14}{13}$ MW3

(ii) converges and oscillates   MW2   5

2 **(i)** centre $(0, -2)$ MW2

radius $= \sqrt{g^2 + f^2 - c}$ M1

$= \sqrt{25}$

$= 5$ W1

(ii) gradient of radius $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2+2}{3-0} = \dfrac{4}{3}$ M1W1

$\therefore$ gradient of tangent $= \dfrac{-3}{4}$ MW1   7

3 (i) Area of sector $\dfrac{1}{2}r^2\theta$ M1

$$= \frac{1}{2} \times 9 \times \frac{\pi}{4}$$

$$= \frac{9\pi}{8}\ \text{cm}^2$$ W1

(ii) Area of triangle $\dfrac{1}{2}ab\sin c$ M1

Area of triangle $= \dfrac{1}{2} \times 1 \times 1 \times \sin 60°$ W1

$= 0.433\ \text{cm}^2$ W1

$\therefore$ Volume of gold $= \left(\dfrac{9\pi}{8} - 0.433\right) \times 0.1$ M1

$= 0.310\ \text{cm}^3$ MW1   **7**

4 (i) $a = 5$

$ar = 3$ $\dfrac{x}{3} = \dfrac{3}{5}$ M1

$\therefore r = \dfrac{3}{5}$ $x = 3 \times \dfrac{3}{5} = \dfrac{9}{5}$ W1

$x = \dfrac{9}{5}$ MW1

(ii) $|r| < 1$ MW1

(iii) $S_\infty = \dfrac{a}{1-r}$ M1

$= 5 \div \dfrac{2}{5}$

$= 12\dfrac{1}{2}$ W1   **6**

5 (i) $(1 + 3x)^4$

$$= 1 + 4 \times 3x + \frac{4 \times 3}{1 \times 2}(3x)^2 + \frac{4 \times 3 \times 2}{1 \times 2 \times 3}(3x)^3 \qquad \text{MW3}$$

$$= 1 + 12x + 54x^2 + 108x^3 \qquad \text{W1}$$

(ii) $1 + 12x + 66x^2 = 1 + 12x + 54x^2 + 108x^3 \qquad \text{M1}$

$$0 = 108x^3 - 12x^2 \qquad \text{W1}$$

$$0 = 12x^2(9x - 1)$$

$$\Rightarrow x = \frac{1}{9} \qquad \text{W1} \qquad 7$$

6 **(a)** $\displaystyle\int 3 - x^{-3}\mathrm{d}x$

$$= 3x + \frac{x^{-2}}{2} + c \qquad\qquad\qquad\text{MW3}$$

(b) $\qquad\qquad\text{Area} = \displaystyle\int_0^a 4x^2 - x^3\mathrm{d}x \qquad\qquad\text{M2W1}$

$$= \left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_0^a \qquad\qquad\text{MW2}$$

$$= \frac{4a^3}{3} - \frac{a^4}{4} \qquad\qquad\text{W1}$$

if 2 areas equal then $\dfrac{4a^3}{3} - \dfrac{a^4}{4} = 0$ $\qquad\text{M1}$

$$a^3\left[\frac{4}{3} - \frac{a}{4}\right] = 0$$

$$a \neq 0 \text{ or } a = \frac{16}{3} \qquad\qquad\text{MW1}$$

or $\text{Area} = \displaystyle\int_0^4 4x^2 - x^3\mathrm{d}x \qquad\qquad\text{M2W1}$

$$= \left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_0^4 \qquad\qquad\text{MW2}$$

$$= \frac{256}{3} - 64 = 21\frac{1}{3} \qquad\qquad\text{W1}$$

$$\text{Area} = -\displaystyle\int_4^a 4x^2 - x^3\mathrm{d}x \qquad\qquad\text{M1}$$

$$= -\left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_4^a$$

$$= -\left(\frac{4a^3}{3} - \frac{a^4}{4} - 21\frac{1}{3}\right)$$

if 2 areas equal then $21\dfrac{1}{3} = -\dfrac{4a^3}{3} + \dfrac{a^4}{4} + 21\dfrac{1}{3}$

$$\Rightarrow a = \frac{16}{3} \qquad\qquad\text{MW1} \qquad 11$$

7 (a) $3 \sin^2 x + 8 \cos x = 0$

$3(1 - \cos^2 x) + 8 \cos x = 0$ M1

$3 - 3 \cos^2 x + 8 \cos x = 0$ W1

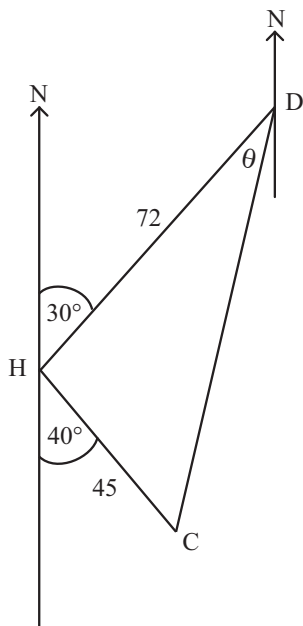$3 \cos^2 x - 8 \cos x - 3 = 0$

$(3 \cos x + 1)(\cos x - 3) = 0$

$\cos x = -\dfrac{1}{3}$ or $\cos x = 3$ MW2

$x = \pm 1.91^c$ no sol$^{n:}$ MW3

(b)

<div style="text-align:right">MW3</div>

Distance $d^2 = 72^2 + 45^2 - 2 \times 72 \times 45 \times \cos 110°$ M1W1

$d = 97.1 \, \text{nm}$ W1

Bearing $\dfrac{\sin \theta}{45} = \dfrac{\sin 110°}{97.1}$ M1W1

$\theta = 25.8°$ W1

bearing $180° + 30° - 25.8° = 184°$ MW1   17

8 (a)

$$3^{2x} = 7$$

$$\log 3^{2x} = \log 7 \qquad \text{M1}$$

$$2x \log 3 = \log 7 \qquad \text{M1W1}$$

$$2x = \frac{\log 7}{\log 3}$$

$$= 0.886 \qquad \text{W1}$$

(b)

$$\log x + \log x^2 + 2 \log x^3 = 1$$

$$\log x + 2 \log x + 6 \log x = 1 \qquad \text{M1W1}$$

$$\log x = \frac{1}{9} \qquad \text{MW1}$$

$$x = 1.29 \qquad \text{M1W1}$$

$$\text{or } \log x + \log x^2 + \log x^6 = 1 \qquad \text{M1W1}$$

$$\log x^9 = 1 \qquad \text{MW1}$$

$$x^9 = 10$$

$$x = 1.29 \qquad \text{M1W1}$$

(c)

$$\log_2 x - \log_2 y = 6$$

$$\log_2 \frac{x}{y} = 6 \qquad \text{M1W1}$$

$$2^6 = \frac{x}{y} \qquad \text{MW1}$$

$$2^3 \times 2^3 = \frac{x}{y} \qquad \text{M1}$$

$$\frac{1}{z^2} = \frac{x}{y} \qquad \text{MW1}$$

$$y = xz^2 \qquad \text{W1} \qquad 15$$

Total		**75**

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2010

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

THURSDAY 24 JUNE, MORNING

MARK
SCHEME

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1 (i) $|\mathbf{A} - \lambda\mathbf{I}| = 0$ M1

$$\Rightarrow \begin{vmatrix} 2-\lambda & 7 \\ 3 & -2-\lambda \end{vmatrix} = 0$$ MW1

$\Rightarrow (2-\lambda)(-2-\lambda) - 21 = 0$ M1
$\Rightarrow -4 + 2\lambda - 2\lambda + \lambda^2 - 21 = 0$
$\Rightarrow \lambda^2 - 25 = 0$ W1
$\Rightarrow (\lambda - 5)(\lambda + 5) = 0$
$\Rightarrow \lambda = \pm 5$ W1

(ii) $\begin{pmatrix} 2 & 7 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 5 \begin{pmatrix} x \\ y \end{pmatrix}$ M1

which gives $\begin{aligned} 2x + 7y = 5x \\ 3x - 2y = 5y \end{aligned}$ $\begin{aligned} \Rightarrow 7y = 3x \\ \Rightarrow 3x = 7y \end{aligned}$ M1

Hence an eigenvector is $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$ W1

Therefore a corresponding unit eigenvector is $\dfrac{1}{\sqrt{58}} \begin{pmatrix} 7 \\ 3 \end{pmatrix}$ MW1 9

2 (i) $\begin{vmatrix} 9 & 2 & a \\ 1 & -1 & -3 \\ a-1 & 1 & 3 \end{vmatrix} = 0$ M1W1

$\Rightarrow 9(-3+3) - 2(3 + 3a - 3) + a(1 + a - 1) = 0$ M1
$\Rightarrow -6a + a^2 = 0$ W1
$\Rightarrow a(-6 + a) = 0$
$\Rightarrow a = 0, 6$ W2

(ii) $\begin{aligned} 9x + 2y + 6z &= 0 & ① \\ x - y - 3z &= 0 & ② \\ 5x + y + 3z &= 0 & ③ \end{aligned}$ M1

② + ③ gives $6x + 0 + 0 = 0$ M1
Hence $x = 0$ W1
Therefore using either ② or ③ gives $y = -3z$ MW1
Check equation ① which gives $0 - 6z + 6z = 0$
The general solution is $(0, -3t, t)$ W1 11

3 **(i)**

°	p	q	r	s	t	u
p	p	q	r	s	t	u
q	q	p	s	r	u	t
r	r	t	p	u	q	s
s	s	u	q	t	p	r
t	t	r	u	p	s	q
u	u	s	t	q	r	p

MW6

(ii) Identity is p   M1
$s^3 = p$ and hence s has period 3   MW1

(iii) Inverse of r is r since $r^2 = p$ M1W1

(iv) {p, s, t}   M1W1   12

4 (i) Centre is (–2, 5)   MW1

Gradient of radius joining centre and the point (2, 7)

$$= \frac{7-5}{2+2} = \frac{1}{2}$$ MW1

The tangent is perpendicular to the radius.
Hence gradient of tangent is –2   MW1
Equation of tangent is $y - 7 = -2(x - 2)$ M1
which gives $y = -2x + 11$ W1

(ii) Substituting the point (3, 5) into the equation of the tangent gives
$5 = -2(3) + 11$
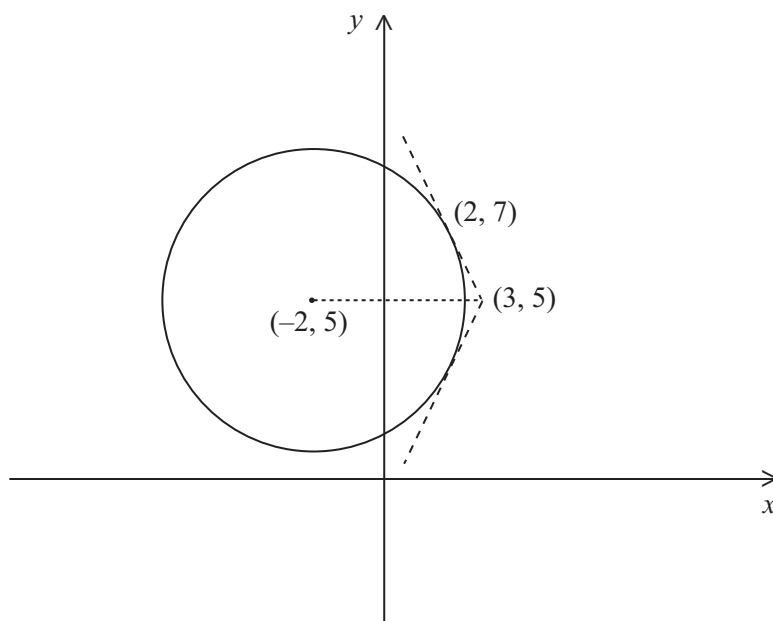$\Rightarrow 5 = -6 + 11$ MW1
Hence (3, 5) lies on the tangent.

(iii)

MW1

By symmetry the gradient of the other tangent is 2.   M1W1
Using the point (3, 5) we obtain
$y - 5 = 2(x - 3)$ M1
$\Rightarrow y = 2x - 6 + 5$
$\Rightarrow y = 2x - 1$ W1   11

5 (a) (i) $\mathbf{M} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ MW2

(ii) Image $= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 7 \\ -3 \end{pmatrix}$ M1

$= \begin{pmatrix} -7 \\ 3 \end{pmatrix}$

Hence image is the point (–7, 3)   W1

(b) (i) Area of T = det $\mathbf{N}$ × Area of S   M1
det $\mathbf{N}$ = –7 – 8 = –15   MW1
Hence area of T = 15 × 3 = 45 cm^2 W1

(ii) $\begin{pmatrix} 1 & 2 \\ 4 & -7 \end{pmatrix}\begin{pmatrix} x \\ mx \end{pmatrix} = \begin{pmatrix} y \\ -my \end{pmatrix}$ M1M1

which gives $x + 2mx = y$
$4x - 7mx = -my$ MW1
Divide to give   M1

$\dfrac{1 + 2m}{4 - 7m} = \dfrac{1}{-m}$

$\Rightarrow -m - 2m^2 = 4 - 7m$ MW1
$\Rightarrow 2m^2 - 6m + 4 = 0$
$\Rightarrow m^2 - 3m + 2 = 0$ W1
Factorise to give $(m - 2)(m - 1) = 0$
Hence $m = 2, 1$ W2   15

6 **(i)** $\left| \sqrt{2} + \sqrt{2}\, i \right| = \sqrt{2+2}$ M1

Modulus = 2   W1

$\left| 1 + \sqrt{3}\, i \right| = \sqrt{1+3}$

Modulus = 2   MW1

$\arg\left(\sqrt{2} + \sqrt{2}\, i \right) = \tan^{-1} \dfrac{\sqrt{2}}{\sqrt{2}}$ M1
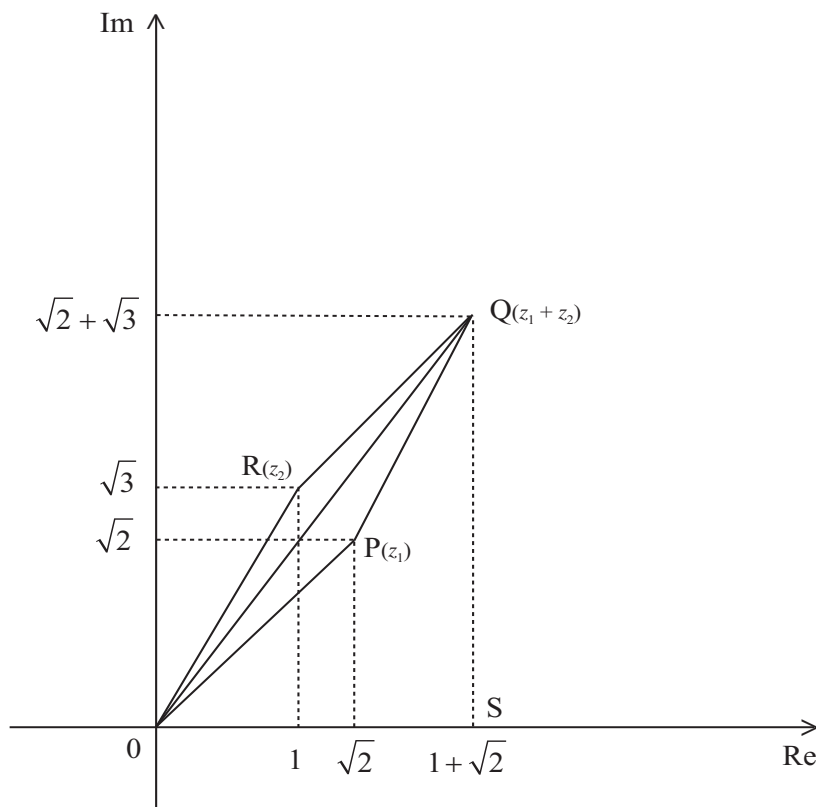
Hence argument $= \dfrac{\pi}{4}$ W1

$\arg\left(1 + \sqrt{3}\, i \right) = \tan^{-1} \dfrac{\sqrt{3}}{1}$
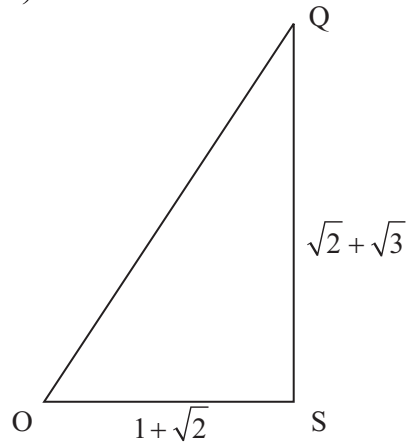
Hence argument $= \dfrac{\pi}{3}$ MW1

(ii)

MW1

MW1

M1W1

MW1

(iii) Angle POR $= \dfrac{\pi}{3} - \dfrac{\pi}{4}$ M1

            $= \dfrac{\pi}{12}$ W1

Angle QOS $= \dfrac{\pi}{4} + \dfrac{1}{2}\left(\dfrac{\pi}{12}\right)$ M1

            $= \dfrac{7\pi}{24}$ W1

Hence

$\tan \dfrac{7\pi}{24} = \dfrac{\sqrt{2}+\sqrt{3}}{1+\sqrt{2}}$ M1W1   17

	Total	**75**

Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2010

Mathematics

Assessment Unit M1
assessing
Module M1: Mechanics 1

[AMM11]

TUESDAY 18 MAY, MORNING

MARK
SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

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1   ↕ $6 \sin 30° = P \cos 10°$ M1W1 M1
          $P = 3.05\,\text{N}$ W1

   ⟷ $Q = P \sin 10° + 6 \cos 30°$ M1W1
        $Q = 5.73\,\text{N}$ W1   **7**

2   (i) $v(\text{m s}^{-1})$
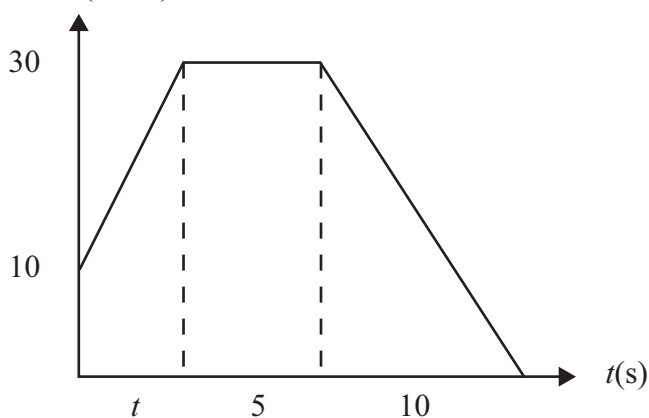
   MW2

(ii) at = change in velocity   M1
          $5 \times t = 30 - 10$
          $t = 4\,\text{s}$ W1

(iii) Total distance = area under graph   M1

$$\frac{10 + 30}{2} \times 4 + \frac{5 + 15}{2} \times 30$$ MW2

$= 380\,\text{m}$ W1   **8**

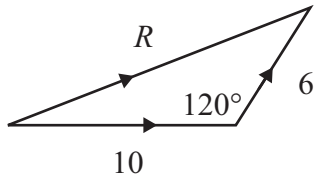**3** **(i)** change in momentum = 0.05 (200–450) M1
$$= -12.5\,\text{Ns}$$ W1

(ii) $I = -F \times 0.002 = -12.5$ M1
$$F = 6250\,\text{N}$$ W1   4

4 **(i)**

$R^2 = 6^2 + 10^2 - 2 \times 6 \times 10 \times \cos 120°$ M1W1
$R = 14\,\text{N}$ W1

or
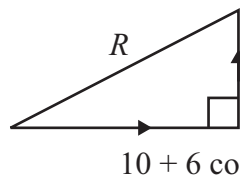
$6 \sin 60°$ M1

$10 + 6 \cos 60°$

$R^2 = (10 + 6\cos 60°)^2 + (6\sin 60°)^2$ W1
$R = 14\,\text{N}$ W1

(ii) $F = ma$ M1
$14 = 4 \times a$
$a = 3.5\,\text{ms}^{-2}$ W1

(iii) $u = 0$

$a = 3.5$ $s = ut + \frac{1}{2}at^2$ M1

$t = 3$ $s = \frac{1}{2} \times 3.5 \times 9$

$s = ?$ $= 15.8\,\text{m}$ W1   7

5 (i)
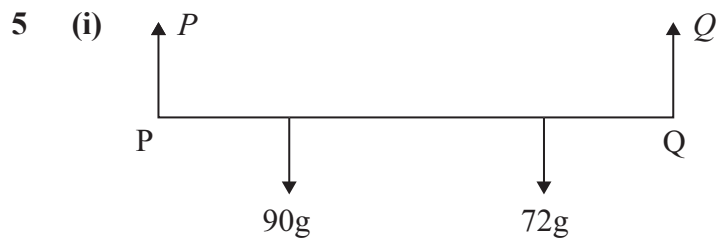
MW2

(ii) $P + Q = 90\text{g} + 72\text{g}$ M1W1

$P + 2P = 162\text{g}$

$P = 54\text{g N (529)}$ MW1

$Q = 108\text{g N (1058)}$ MW1

(iii) Moments about P $\swarrow$ $72\text{g} \times 4 + 90\text{g} \times x = 108\text{g} \times 6$ M2W2

$x = 4\,\text{m}$ W1   11

6 (i) $a = 4t - 10t^3$

$v = \int 4t - 10t^3 \, \mathrm{d}t$ M1

$= 2t^2 - \dfrac{5}{2}t^4 + c$ W1

at $t = 0$ $v = \dfrac{5}{2}$ $\therefore c = \dfrac{5}{2}$ M1W1

$v = 2t^2 - \dfrac{5}{2}t^4 + \dfrac{5}{2}$

(ii) $s = \int 2t^2 - \dfrac{5}{2}t^4 + \dfrac{5}{2} \, \mathrm{d}t$ M1

$= \dfrac{2t^3}{3} - \dfrac{t^5}{2} + \dfrac{5}{2}t + d$ W1

at $t = 0$ $s = 0$ $\therefore d = 0$ MW1

$s = \dfrac{2t^3}{3} - \dfrac{t^5}{2} + \dfrac{5}{2}t$

(iii) when returns to start $s = 0$ M1

$$\frac{2t^3}{3} - \frac{t^5}{2} + \frac{5t}{2} = 0$$

$$4t^3 - 3t^5 + 15t = 0$$

$$3t^5 - 4t^3 - 15t = 0$$

$$t\left(3t^4 - 4t^2 - 15\right) = 0$$ W1

$$t\left(3t^2 + 5\right)\left(t^2 - 3\right) = 0$$ W1

$$t = 0 \quad \text{no sol}^n \quad t^2 = 3$$

$$\therefore t = \sqrt{3} \text{ s}$$

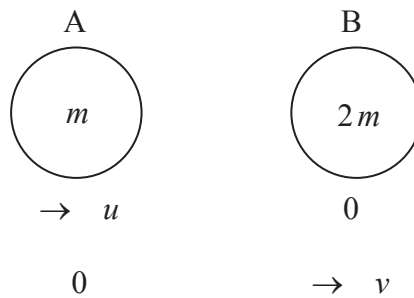returns to start after $\sqrt{3}$ s   W1   11

7   **(i)** $\rightarrow +$

A   B

m $2m$

$\rightarrow u$ 0

0                     $\rightarrow v$

momentum before = momentum after   M1

$$mu + 0 = 0 + 2mV$$ MW2

$$V = \frac{u}{2}$$ W1

(ii) → +

B   C

$2\,m$ $3\,m$

→ $\dfrac{u}{2}$ 0

→ v $\dfrac{2u}{3}$
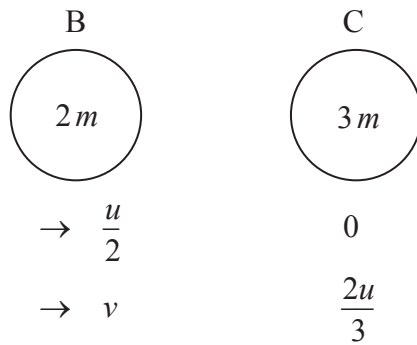
momentum before = momentum after

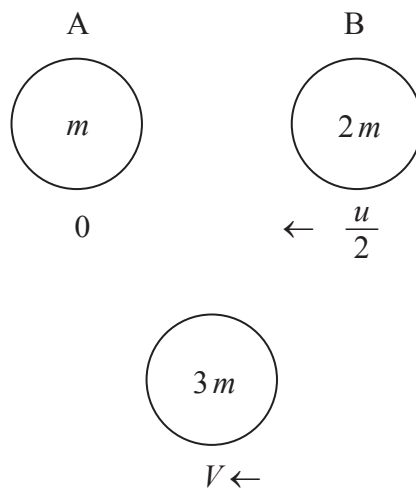$$2m \times \frac{u}{2} + 0 = 2m \times V + 3m \times \frac{2u}{3}$$ MW2

$$u = 2V + 2u$$ W1

$$V = \frac{-u}{2}$$

∴ B will collide with A again as B now moving towards A which is at rest.   W1

(iii) + ←

A   B

m $2\,m$

0   ← $\dfrac{u}{2}$

$3\,m$

$V \leftarrow$

momentum before = momentum after

$$2m \times \frac{u}{2} + 0 = 3m \times V$$ MW1
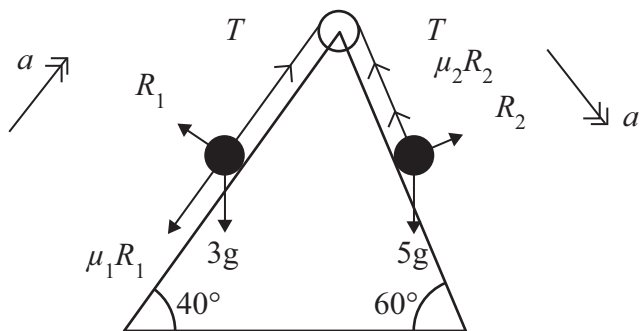
$$V = \frac{u}{3}$$ W1

Speed $\dfrac{u}{3}$ in opposite direction to initial direction of A   W1   11

8 (i)

MW3

(ii) P $F = ma$ M1

$T - \mu_1 R_1 - 3g \sin 40° = 3a$ M1W2

Q $F = ma$

$5g \sin 60° - T - \mu_2 R_2 = 5a$ MW2

$R_1 = 3g \cos 40°$ M1W1
$R_2 = 5g \cos 60°$ MW1

   $T - 0.3 \times 3g \cos 40° - 3g \sin 40° = 3a$
   $5g \sin 60° - T - 0.15g \cos 60° = 5a$ M1

$5g \sin 60° - 0.9g \cos 40° - 3g \sin 40° - 0.5g \cos 60° = 8a$

          $a = 1.79 \, \text{m s}^{-2}$ W2

$T - 0.9g \cos 40° - 3g \sin 40° = 3 \times 1.79$
        $T = 31.0 \, \text{N}$ MW1   16

             Total   **75**

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
Summer 2010

Mathematics

Assessment Unit 1

assessing

Module S1: Statistics 1

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1 Midvalues: 24.5, 74.5, 124.5, 174.5, 224.5, (275)   M1

From calculator $n = 200$ $\sum fx = 27300$ $\sum fx^2 = 4412650$

$\bar{x} = 136.5$ M1 W1

$\sigma_n = 58.574\ldots = 58.6$ (3sf)   M1 W1   5

2 **(i)** The outcome of one trial does not influence another   M1

(ii) • 2 possible outcomes   M1

• probability of success is constant   M1

(iii) Let X be r.v. "the number of people carrying a donor card"

$X \sim B\,(6, 0.35)$

$P(X = 3) = \binom{6}{3}(0.35)^3(0.65)^3 = 0.235$ (3sf)   M1 W1

(iv) $P(X \geq 3) = 1 - P(X < 3)$ M1

$= 1 - P(X = 0, 1, 2)$

$= 1 - [\binom{6}{0}(0.35)^0(0.65)^6 + \binom{6}{1}(0.35)^1(0.65)^5$

$+ \binom{6}{2}(0.35)^2(0.65)^4]$ MW3

$= 1 - [0.0754\ldots + 0.24366\ldots + 0.32800\ldots]$

$= 0.3529\ldots$

$= 0.353$ (3s.f.)   W1   10

3 **(i)** Let X be r.v. "No. of misprints per page"

$X \sim$ Po (2.6)   M1

$$P(X = 2) = \frac{e^{-2.6} \times 2.6^2}{2!} = 0.251 \text{ (3s.f.)}$$ M1 W1

(ii) $P(X \geq 3) = 1 - P(X < 3)$ M1

$$= 1 - P(X = 0, 1, 2)$$

$$= 1 - e^{-2.6} \left[\frac{2.6^0}{0!} + \frac{2.6^1}{1!} + \frac{2.6^2}{2!} \right]$$ MW2

$$= 1 - 6.98e^{-2.6}$$

$$= 0.482 \text{ (3sf)}$$ W1

(iii) $P(X = 5 \mid X \geq 3) = \dfrac{P(X = 5 \cap X \geq 3)}{P(X \geq 3)}$ M1

$$= \frac{P(X = 5)}{P(X \geq 3)}$$ MW1

$$P(X = 5) = \frac{e^{-2.6} \times 2.6^5}{5!} = 0.0735$$ MW1

$$P(X = 5 \mid X \geq 3) = \frac{0.0735}{0.482} = 0.153 \text{ (3s.f.)}$$ W1   11

4 **(i)**

x	3	5	7	9	11	13	15	
$P(X = x)$	$\dfrac{1}{16}$	$\dfrac{2}{16}$	$\dfrac{3}{16}$	$\dfrac{4}{16}$	$\dfrac{3}{16}$	$\dfrac{2}{16}$	$\dfrac{1}{16}$	MW4

(ii) Distribution is symmetrical. 9 is central value of X M2

(iii) $E(X^2) = 3^2 \times \dfrac{1}{16} + 5^2 \times \dfrac{2}{16} + 7^2 \times \dfrac{3}{16} + 9^2 \times \dfrac{4}{16} + 11^2 \times \dfrac{3}{16}$ M1

$$+ 13^2 \times \frac{2}{16} + 15^2 \times \frac{1}{16} = 91$$ W1

$$\text{Var} (X) = E(X^2) - [E(X)]^2$$ M1

$$= 91 - 9^2 = 10$$ W1   10

5 (i) $\int_0^1 k(x - x^3)\,dx = 1$ $\qquad$ M2

$k\left[\dfrac{x^2}{2} - \dfrac{x^4}{4}\right]_0^1 = 1$ $\qquad$ W1

$k\left(\dfrac{1}{2} - \dfrac{1}{4}\right) = 1 \longrightarrow \quad k = 4$ $\qquad$ W1

(ii) $P(0 \leq X \leq \dfrac{1}{2}) = \int_0^{\frac{1}{2}} (4x - 4x^3)\,dx$ $\qquad$ M1

$= [2x^2 - x^4]_0^{\frac{1}{2}}$ $\qquad$ MW1

$= \dfrac{1}{2} - \dfrac{1}{16} = \dfrac{7}{16}$ $\qquad$ W1

(iii) $E(X^2) = \int_0^1 x^2(4x - 4x^3)\,dx = \int_0^1 (4x^3 - 4x^5)\,dx$ $\qquad$ M1

$= \left[x^4 - \dfrac{2}{3}x^6\right]_0^1 = \left(1 - \dfrac{2}{3}\right) - (0)$ $\qquad$ W1

$= \dfrac{1}{3}$ $\qquad$ W1

$\text{Var}(X) = E(X^2) - [E(X)]^2$ $\qquad$ M1

$= \dfrac{1}{3} - \left(\dfrac{8}{15}\right)^2 = \dfrac{11}{225}$ (0.048) $\qquad$ W1 $\qquad$ 12

6 **(i)** $X \sim N(4000, \sigma^2)$

$P(X > 4035) = 0.2420$

$\longrightarrow P(X < 4035) = 0.7580$ MW1

$\longrightarrow P\left(Z < \dfrac{4035 - 4000}{\sigma}\right) = 0.7580$ M1

$\dfrac{35}{\sigma} = \Phi^{-1}(0.7580)$

$\dfrac{35}{\sigma} = 0.7$ MW1 W1

$\sigma = 50$ W1

(ii) $P(X > 4110) = P\left(Z > \dfrac{4110 - 4000}{50}\right) = P(Z > 2.2)$ MW1

$= 1 - \Phi(2.2)$

$= 1 - 0.9861$ MW1

$= 0.0139$ W1

$P(X < 3888) = P\left(Z < \dfrac{3888 - 4000}{50}\right) = P(Z < -2.24)$ MW1

$= 1 - \Phi(2.24)$

$= 1 - 0.9875$ MW1

$= 0.0125$ W1

$P(X > 4110 \text{ or } X < 3888) = 0.0139 + 0.0125 = 0.0264$ M1

$= 2.64\%$ W1   13

7 **(a)** **(i)** The events can't both happen   M1

        $P(A \cap B) = 0$ M1

    (ii) It is certain that at least one event occurs   M1

        $P(A \cup B) = 1$ M1

(b) $P(G) = 0.4$ $P(F) = 0.35$ $P(G \cup F) = 0.61$

    $P(F \cap G)$ $= P(G) + P(F) - P(G \cup F)$ M1

               $= 0.4 + 0.35 - 0.61 = 0.14$ W1

    $P(G) \times P(F)$ $= 0.4 \times 0.35 = 0.14 = P(F \cap G)$ M1

    Hence **Yes** the events are independent   M1

(c) **(i)** $P(D) = x$ $P(W|D) = 0.75$ $P(W \cup D) = 0.36$

        $P(W \cap D)$ $= P(D) \times P(W|D) = 0.75x$ M1 W1

        $P(W \cup D)$ $= P(W) + P(D) - P(W \cap D)$ M1

        0.36 $= P(W) + x - 0.75x$

        $\longrightarrow P(W) = 0.36 - 0.25x$ W1

    (ii) $P(W) = 1.25x$

        $1.25x = 0.36 - 0.25x$ MW1

        $1.5x = 0.36$

        $x = 0.24$ W1   14

                                    Total   **75**