Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2011

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

AMF11

[AMF11]

WEDNESDAY 19 JANUARY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 7 & -4 \\ -2 & 5 \end{pmatrix}$$

(i) Show that the eigenvalues of **A** are 3 and 9 [5]

(ii) Find a **unit** eigenvector corresponding to the eigenvalue 9 [4]

2 Two circles have equations

$$x^2 + y^2 + 2x - 6y + 8 = 0$$

$$x^2 + y^2 - 4x - 28 = 0$$

(i) Find the point where these circles meet. [8]

(ii) Determine whether the circles touch internally or externally. [4]

3 **(a)** Explain why the set {1, 2, 3, 4, 5, 6, 7} cannot form a group under **multiplication** modulo 8 [3]

(b) **(i)** Copy and complete the group table for **addition** modulo 8

	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5					
4	4	5	6					
5	5	6	7					
6	6	7	0					
7	7	0	1					

[5]

(ii) Using the group table in **(i)**, or otherwise, write down the two values of x which satisfy

$$x^3 = x$$ [2]

(iii) For this group, write down a subgroup of order 4 [3]

4 **(a)** Describe fully the transformation given by the matrix

$$\mathbf{M} = \begin{pmatrix} -\frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

[5]

(b) The set of points which form the circle

$$x^2 + y^2 = 25$$

is mapped under a transformation given by the matrix

$$\mathbf{N} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$$

Show that the equation of the curve formed by the image points is

$$5X^2 + 13Y^2 - 16XY = 25$$

[8]

5 A matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 3 & 2 & a \\ 1 & -2 & -1 \\ a & 0 & 3 \end{pmatrix}$$

(i) Find, in terms of a, the determinant of **M**. [3]

A system of linear equations is given by

$$\begin{aligned} 3x + 2y + az &= 7 \\ x - 2y - z &= 1 \\ ax + 3z &= 11 \end{aligned}$$

(ii) Find the values of a for which the system has a unique solution. [3]

(iii) If $a = 1$, find the inverse of **M**. [6]

(iv) Hence, for $a = 1$ find the unique solution of the system of equations. [3]

6 **(a)** Find the complex roots of the equation

$$2z^2 - 2iz - 5 = 0$$ [4]

(b) **(i)** Sketch, on an Argand diagram, the locus of those points w which satisfy

$$|w - 3| = 5$$ [3]

(ii) On the same diagram, shade the region which represents the locus of those points w which satisfy

$$|w - 3| \le 5$$

and

$$\frac{\pi}{6} \le \arg(w - 3) \le \frac{\pi}{4}$$ [6]

THIS IS THE END OF THE QUESTION PAPER