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ADVANCED
General Certificate of Education
January 2011

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

AMC41

FRIDAY 28 JANUARY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(a)** **(i)** Write

$$\frac{1}{(x+1)(x-1)}$$

in partial fractions. [4]

(ii) Hence find

$$\int \frac{1}{(x+1)(x-1)}\, dx$$ [4]

(b) Use integration by parts to find

$$\int x \cos x \, dx$$ [5]

2 The vector equation of the line PQ is

$$\mathbf{r} = (2\mathbf{i} + 3\mathbf{j}) + \mu(\mathbf{i} - \mathbf{j} - 2\mathbf{k}).$$

The line PR has direction vector

$$(4\mathbf{i} + 5\mathbf{j} + \mathbf{k})$$

Find the angle between the lines PQ and PR. [7]

3 Solve the equation

$$\cos x = 2 \sin (x + 60^\circ)$$

for $-180^\circ \leqslant x \leqslant 180^\circ$ [7]

4 A curve is given parametrically as

$$x = \cot\theta \qquad y = \operatorname{cosec}\theta$$

(i) Show that the cartesian equation of the curve is

$$y^2 = 1 + x^2$$ [2]

(ii) Hence find the **exact** gradients of the tangents to the curve at the points where $x = 1$ [7]

5 **(i)** Show that

$$\tan x \sec^4 x \equiv \tan x \sec^2 x + \tan^3 x \sec^2 x$$ [3]

(ii) Hence, and by using the substitution $u = \tan x$, or otherwise, find

$$\int_0^{\frac{\pi}{4}} \tan x \ \sec^4 x \ \mathrm{d}x$$ [7]

6 Solve the differential equation

$$(1 + x^2)\ \frac{\mathrm{d}y}{\mathrm{d}x} = x(1 + y)$$

to find y in terms of x, given that $x = 0$ when $y = 0$ [11]

7 The bowl of a glass can be modelled by the rotation of the curve

$$y = e^{\frac{x}{6}}$$

between $x = 0$ and $x = 9$ cm, as shown in **Fig. 1** below, through 2π radians about the x-axis.

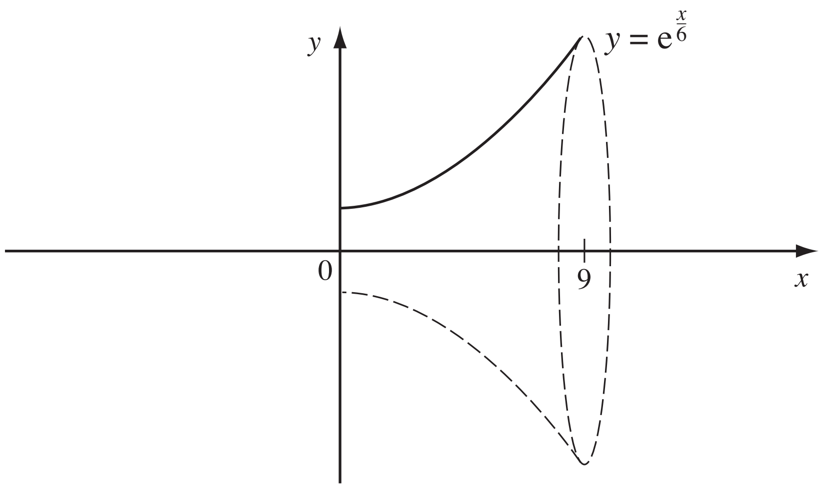

Fig. 1

Find the maximum volume that the glass can hold. [7]

8 The function f is defined as

$$f : x \to \sin x \qquad \text{for } -\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$$

(i) Write down the inverse function f^{-1} and state its domain and range. [4]

The function g is defined as

$$g : x \to |x| \qquad \text{for } x \in \mathbb{R}$$

(ii) Find the composite function gf, stating its range. [4]

(iii) Hence sketch the graph of

$$y = gf(x)$$ [3]