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**ADVANCED**
**General Certificate of Education**
**2011**

# Mathematics

Assessment Unit M3

*assessing*

Module M3: Mechanics 3

**[AMM31]**

**MONDAY 20 JUNE, MORNING**

AMM31

## TIME

1 hour 30 minutes.

## INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

## INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,m\,s^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

**Answer all six questions.**

**Show clearly the full development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

1 A particle P of mass 1 kg moves from the origin O to a point A through a zero gravity zone. The displacement of A from O is **s** metres where

$$\mathbf{s} = 6\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$$

P is acted on by two forces $\mathbf{F_1}$ newtons and $\mathbf{F_2}$ newtons where

$$\mathbf{F_1} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \qquad \mathbf{F_2} = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$$

$W_1$ is the work done by $\mathbf{F_1}$ in moving the particle from O to A and $W_2$ is the work done by $\mathbf{F_2}$ in moving the particle from O to A.

**(i)** Find **(a)** $W_1$

**(b)** $W_2$ [4]

P passes through O when $t = 0$ s with initial velocity **u** m s$^{-1}$ where

$$\mathbf{u} = \begin{pmatrix} 6 \\ 2 \\ -4 \end{pmatrix}$$

and reaches A when $t = 2$ s.

**(ii)** Find the **velocity** of P at A. [4]

**(iii)** Verify that the total work done on the particle is 84 J and that this satisfies the Work–Energy Principle. [4]

**2** [Take **i** to be a unit vector direction East and **j** to be a unit vector direction North.]

A burglar is running along a straight path with velocity $(4\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$
A police sergeant is 120 m East and 80 m South of the burglar's position when she spots him.
At time $t = 0$ s she starts running with velocity $(\mathbf{i} + 5\mathbf{j})\,\text{m s}^{-1}$

**(i)** Find the velocity of the sergeant relative to the burglar. [3]

**(ii)** Show that the sergeant will catch the burglar and find the time at which this will occur. [6]

**3** An elastic string is attached to a fixed point A and hangs vertically in equilibrium supporting a particle of mass 0.1 kg.

The string is of natural length 0.5 m and modulus of elasticity 1.8 N. The extension in the string is $\frac{49}{180}$ m.

Initially the particle is pulled vertically downwards a further distance of 0.2 m and released from rest.

After $t$ seconds the particle is $x$ metres below the equilibrium position as shown in **Fig. 1** below.

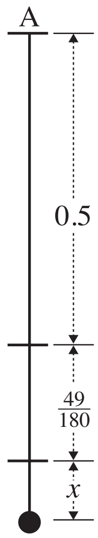


**Fig. 1**

**(i)** Show that the equation of motion of the particle is

$$\ddot{x} = -36x$$

and that this represents S.H.M. [6]

**(ii)** Find the value of $t$ when the particle is first 0.1 m above the equilibrium position. [7]

**4** A variable force

$$F = 15 + 12x - 3x^2$$

acts on a particle P of mass $\frac{8}{9}$ kg as it moves along a smooth, straight, horizontal track, where $x$ is P's distance from a fixed point O.
**Fig. 2** below shows the graph of $F$.

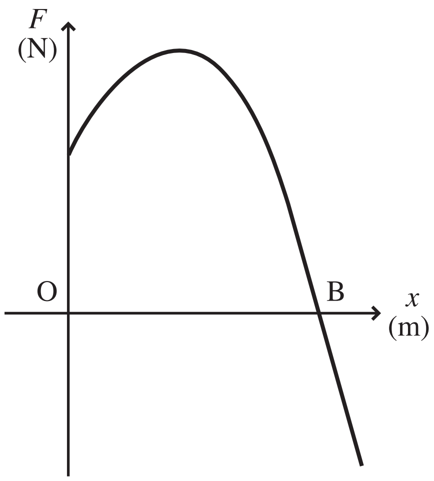


**Fig. 2**

The graph crosses the $x$-axis at B.

**(i)** Find the work done by $F$ as it moves the particle from O to B. [7]

The particle passes through O with a speed of $8\,\text{m}\,\text{s}^{-1}$

**(ii)** Use the Work–Energy Principle to find P's maximum speed. [4]

**5** **Fig. 3** below shows a circular steel plate that forms one end of a storage cylinder.
The plate has a radius of 0.5 m.
Its centre is at C where the $x$ and $y$-axes meet.
Two discs to make holes for inspection hatches are cut in the plate.
One disc of radius 0.1 m centre A (0, 0.3).
The other disc of radius 0.2 m centre B (0.2, –0.2).

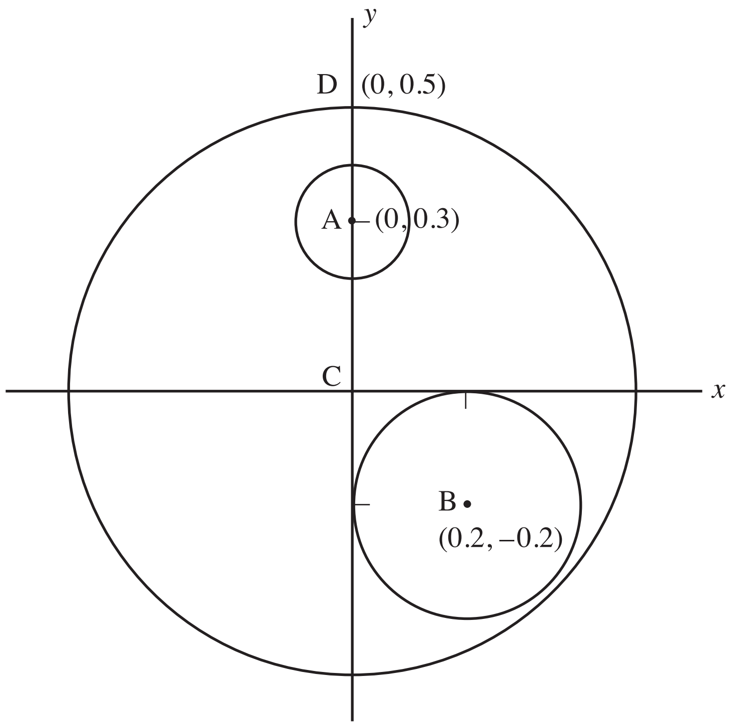


**Fig. 3**

Model this plate as a lamina of uniform density.
The mass of steel removed from the smaller hole is $m$ kg.

**(i)** Show that when both discs have been removed from the plate its mass is then $20m$ kg. [3]

The centre of mass of the plate is now at G $(\bar{x}, \bar{y})$.

**(ii)** Show that $\bar{x} = -0.04$ m and find $\bar{y}$. [8]

The plate has an actual mass of 20 kg and rests on rough ground with A vertically above C.
It is kept in equilibrium by a force $P$ acting at D (0, 0.5) and parallel to the $x$-axis.

**(iii)** Find $P$. [4]

**6** A particle B of mass $m$ kg is attached to two springs $S_1$ and $S_2$ whose other ends are attached to two fixed points A and C respectively.
A is at the bottom and C at the top of a line of greatest slope of a smooth plane inclined at $\alpha$ to the horizontal as shown in **Fig. 4** below.

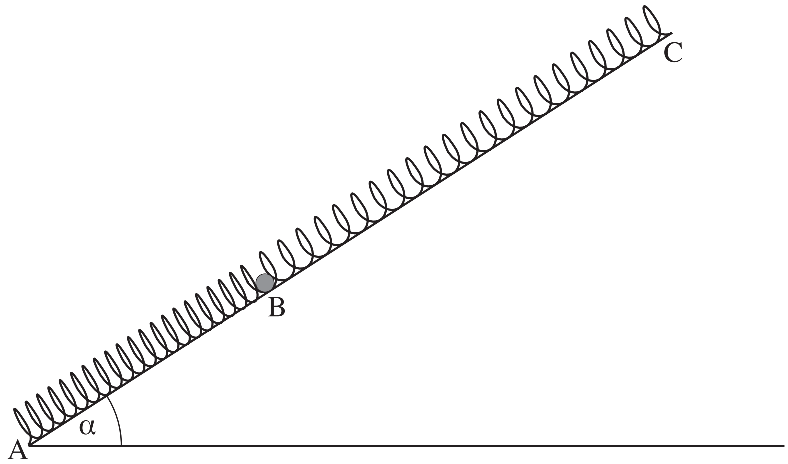


**Fig. 4**

Both springs are of natural length $l$ metres and modulus of elasticity $0.5mg$ newtons.
$AC = 2.5l$ metres
The system rests in equilibrium with $S_1$ extended $x$ metres and $S_2$ extended $y$ metres.

**(i)** Show that $x + y = 0.5l$ [2]

**(ii)** By considering the forces acting on B, find $x$ in terms of $l$ and $\alpha$. [7]

**(iii)** If $S_1$ is compressed show that

$$\sin\alpha > 0.25$$ [2]

**(iv)** If $\sin\alpha = 0.75$, find the elastic energy stored in $S_1$ in terms of $l$, $m$ and g. [4]

**THIS IS THE END OF THE QUESTION PAPER**

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