Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2012

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

AMF11

[AMF11]

FRIDAY 20 JANUARY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Let $\mathbf{M} = \begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(i) Show that $\mathbf{M}^2 = 2\mathbf{M} - 3\mathbf{I}$ [4]

(ii) Hence, or otherwise, express the matrix $\mathbf{M}^3$ in the form $\alpha\mathbf{M} + \beta\mathbf{I}$, where α and β are integers. [4]

2 The matrix **M** is given by

$$\begin{pmatrix} 7 & 3 \\ 3 & -1 \end{pmatrix}$$

(i) Show that the two eigenvalues of **M** are 8 and -2 [5]

(ii) For each eigenvalue find a corresponding unit eigenvector. [7]

P is a 2×2 matrix such that

$$\mathbf{P}^{-1}\mathbf{M}\mathbf{P} = \mathbf{D}$$

where **D** is a diagonal matrix.

(iii) Write down a possible matrix **P**. [1]

3 A binary operation $*$ is defined for all real numbers a and b.
The operation is given as

$$a * b = a + b - 7$$

(i) Show that $*$ is associative. [4]

(ii) Find the identity element. [3]

(iii) Find the inverse of the element a. [3]

(iv) Determine whether the set of all real numbers forms a group under the operation $*$
Give clear reasons for your answer. [2]

4 **(a)** **(i)** The shear represented by the matrix **S** maps the points (3, 4) and (7, 1) onto (10, −3) and (15, −7) respectively.
Find the matrix **S**. [5]

(ii) The shear represented by the matrix **S** maps a region P of area 12 cm^2 to a new region Q.
Find the area of Q. [3]

(b) **(i)** Describe the difference between an invariant line and a line of invariant points under a linear transformation. [2]

(ii) The line $y = mx$ is a line of invariant points under the transformation represented by the matrix

$$\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$$

Find the value of m. [5]

5 The equation of a circle is given by

$$x^2 + y^2 - 2x - 4y = 0$$

(i) Find the equation of the tangent to the circle at the point (3, 3). [6]

(ii) Verify that this tangent passes through the point (1, 7). [1]

(iii) Hence, or otherwise, find the equation of the other tangent to the circle from the point (1, 7). [5]

6 **(a)** Find all real values of a and b such that

$$(a + b\mathrm{i})^2 = -16 - 30\mathrm{i}$$ [8]

(b) **(i)** Sketch on an Argand diagram the locus of those points z which satisfy

$$|z - (2 + 4\mathrm{i})| = \sqrt{5}$$ [3]

(ii) Hence, or otherwise, find the maximum value of $|z|$ for any point z which lies on this locus. [4]