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ADVANCED
General Certificate of Education
January 2012

Mathematics

Assessment Unit M2

assessing

Module M2: Mechanics 2

[AMM21]

MONDAY 30 JANUARY, MORNING

AMM21

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answer should be given to three significant figures unless otherwise stated.

1 At time $t = 0$ seconds a particle is at a fixed point O and has an initial velocity of $(\mathbf{i} + 3\mathbf{j})\,\mathrm{m\,s^{-1}}$
The constant acceleration of the particle is $(4\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-2}}$

(i) Find the velocity of the particle when $t = 3$ [3]

(ii) Find the **distance** of the particle from O when $t = 2$ [5]

2 A small box B of mass m kg is placed on a rough horizontal rotating disc. B is r metres from the centre of rotation as shown in **Fig. 1** below.

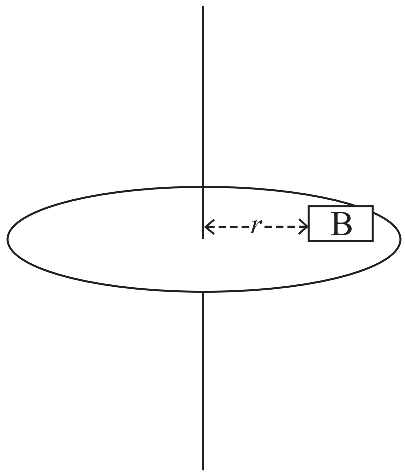

Fig. 1

The coefficient of friction between the disc and B is μ.

Find, in terms of μ, r and g, the maximum speed v that can be given to B without it slipping. [7]

3 **Fig. 2** below shows a smooth bead of mass m kg threaded onto a smooth wire in the shape of a circle. The wire is fixed in a vertical plane.

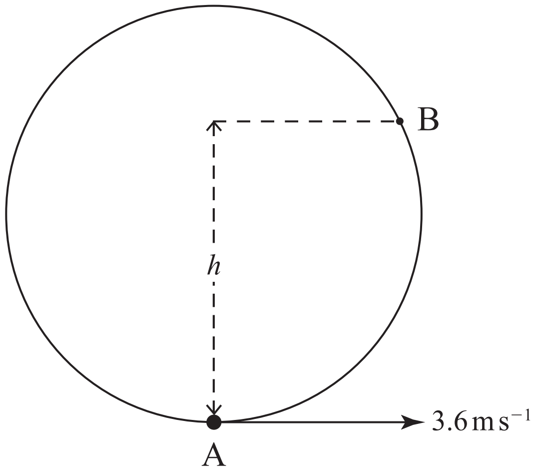

Fig. 2

The bead is resting at the lowest point A when it is given an initial velocity of $3.6\,\text{m s}^{-1}$
The bead first comes to rest at a point B which is h metres vertically above A.

(i) Find, in terms of m, the kinetic energy of the bead at A. [2]

(ii) Find, in terms of m and h, the potential energy of the bead at B. [2]

(iii) Use the conservation of mechanical energy to find h. [3]

(iv) Briefly explain why it is possible to use the conservation of mechanical energy to answer part **(iii)**. [1]

4 A particle, P, moves so that its displacement, $\mathbf{r}$ metres, from a fixed point at any time, t seconds, is given by

$$\mathbf{r} = (2t^3 - 3t^2)\mathbf{i} - t^2\mathbf{j} - 2\mathbf{k}$$

(i) Find an expression for the velocity of P at any time t. [3]

(ii) Find the **speed** of P when $t = 2$ [3]

(iii) Find an expression for the acceleration of P at any time t. [2]

P has a mass of 3 kg.

(iv) Find the force acting on P when $t = 1$ [3]

(v) Find the direction in which the force found in **(iv)** is acting. [3]

5 A car of mass 900 kg is travelling along a horizontal road against resistances totalling $(3 + 4v)$ N, where v m s^{-1} is the speed of the car.
The engine of the car is working at a steady rate of X watts.

(i) Given that the maximum speed of the car is 40 m s^{-1}, find X. [6]

(ii) Find the acceleration of the car when it is travelling at 15 m s^{-1} [4]

6 A builder is reroofing a house.
As he removes the tiles he lets them slide down a rigid plastic shute into a skip at the side of the house.
The shute is rough. It is 12 m long and is inclined at an angle of 30° to the horizontal as shown in **Fig. 3** below.

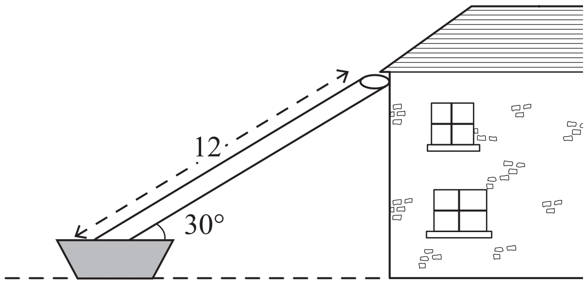

Fig. 3

The builder pushes a tile of mass 2.5 kg down the shute giving it an initial speed of 2 m s^{-1}
The coefficient of friction between the tile and the shute is 0.2

(i) Draw a diagram showing the external forces acting on the tile. [2]

(ii) Find the work done against friction. [4]

(iii) Using the work-energy principle, find the speed with which the tile enters the skip. [6]

7 A particle is projected from a point O on a horizontal plane with an initial speed of $u\,\mathrm{m\,s^{-1}}$ and at an angle θ above the plane.

(i) Show that the equation of the flight path of the particle is

$$y = x \tan \theta - \frac{gx^2 \sec^2 \theta}{2u^2}$$

where (x, y) is any point on the particle's trajectory. [6]

A player releases a basketball with a speed of $10\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal. The ball falls into a basket which is 1 m vertically above and 5 m horizontally from the point of projection as shown in **Fig. 4** below.

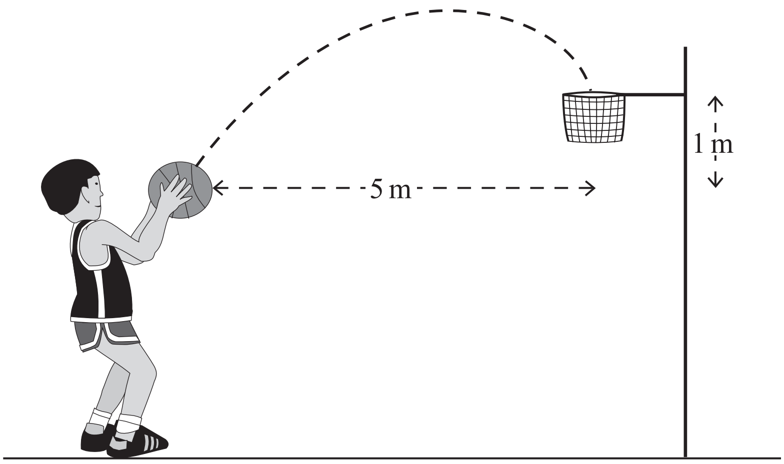

Fig. 4

(ii) Taking g as $10\,\mathrm{m\,s^{-2}}$, show that θ must satisfy the equation

$$5 \tan^2 \theta - 20 \tan \theta + 9 = 0$$ [4]

For the ball to fall through the basket, the angle the flight path has with the vertical at the basket, should be as small as possible.

(iii) Use the equation in **(ii)** to find the value of θ which will make this happen. [5]

(iv) State **one** modelling assumption that you have made. [1]

THIS IS THE END OF THE QUESTION PAPER

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