Rewarding Learning

ADVANCED
General Certificate of Education
January 2012

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

FRIDAY 20 JANUARY, AFTERNOON

MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for correct working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidates's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

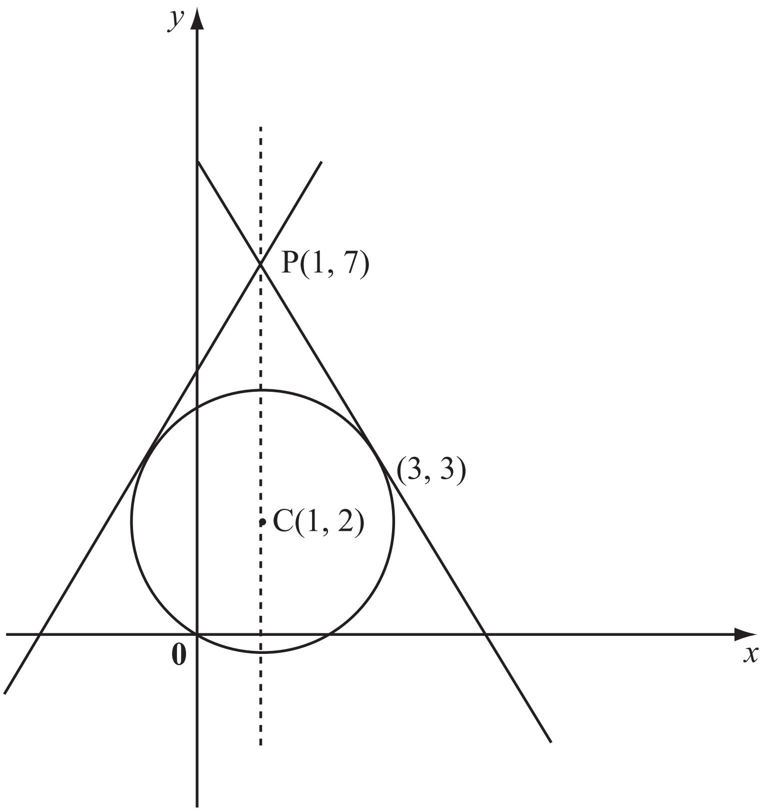
When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

				AVAILABLE MARKS
1	(i)	$\mathbf{M}^2 = \begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}$	M1	
		$\Rightarrow \mathbf{M}^2 = \begin{pmatrix} 1 & -2 \\ 6 & -3 \end{pmatrix}$	W1	
		Also: $2\mathbf{M} - 3\mathbf{I} = \begin{pmatrix} 4 & -2 \\ 6 & 0 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	M1	
		$\Rightarrow 2\mathbf{M} - 3\mathbf{I} = \begin{pmatrix} 1 & -2 \\ 6 & -3 \end{pmatrix}$	W1	
		Therefore $\mathbf{M}^2 = 2\mathbf{M} - 3\mathbf{I}$		
	(ii)	$\mathbf{M}^3 = \mathbf{M}(2\mathbf{M} - 3\mathbf{I})$	M1	
		$\Rightarrow \mathbf{M}^3 = 2\mathbf{M}^2 - 3\mathbf{M}$	W1	
		$\Rightarrow \mathbf{M}^3 = 2(2\mathbf{M} - 3\mathbf{I}) - 3\mathbf{M}$	M1	
		$\Rightarrow \mathbf{M}^3 = 4\mathbf{M} - 6\mathbf{I} - 3\mathbf{M}$		
		$\Rightarrow \mathbf{M}^3 = \mathbf{M} - 6\mathbf{I}$	W1	8
2	(i)	$\det(\mathbf{M} - \lambda\mathbf{I}) = 0$	M1	
		$\Rightarrow \begin{vmatrix} 7-\lambda & 3 \\ 3 & -1-\lambda \end{vmatrix} = 0$	MW1	
		$\Rightarrow (7-\lambda)(-1-\lambda) - 9 = 0$	M1	
		$\Rightarrow -7 + \lambda - 7\lambda + \lambda^2 - 9 = 0$		
		$\Rightarrow \lambda^2 - 6\lambda - 16 = 0$	W1	
		$\Rightarrow (\lambda - 8)(\lambda + 2) = 0$		
		$\Rightarrow \lambda = 8, -2$	W1	
		Therefore the eigenvalues are 8 and –2		
	(ii)	For $\lambda = 8$, then $\begin{pmatrix} 7 & 3 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 8\begin{pmatrix} x \\ y \end{pmatrix}$	M1	
		Hence $7x + 3y = 8x \quad \Rightarrow x = 3y$ and $3x - y = 8y \quad \Rightarrow 3x = 9y$	M1	
		Therefore one possible eigenvector could be $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$	W1	
		The corresponding unit eigenvector is $\frac{1}{\sqrt{10}}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$	W1	
		For $\lambda = -2$, then $\begin{pmatrix} 7 & 3 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -2\begin{pmatrix} x \\ y \end{pmatrix}$	MW1	
		Hence $7x + 3y = -2x \quad \Rightarrow 9x = -3y$ and $3x - y = -2y \quad \Rightarrow 3x = -y$		
		Therefore one possible eigenvector could be $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$	MW1	
		The corresponding unit eigenvector is $\frac{1}{\sqrt{10}}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$	MW1	
	(iii)	A possible matrix **P** is $\frac{1}{\sqrt{10}}\begin{pmatrix} 3 & 1 \\ 1 & -3 \end{pmatrix}$	MW1	13

				AVAILABLE MARKS
3	(i)	$a * (b * c) = a * (b + c - 7)$	M1	
		$= a + b + c - 7 - 7$		
		$= a + b + c - 14$	W1	
		Similarly,		
		$(a * b) * c = (a + b - 7) * c$	M1	
		$= a + b - 7 + c - 7$		
		$= a + b + c - 14$	W1	
		Since $a * (b * c) = (a * b) * c$, then the operation $*$ is associative		
	(ii)	If I is the identity element, then $a * I = a$	M1	
		Hence $a + I - 7 = a$	M1	
		$\Rightarrow I = 7$	W1	
		Therefore the identity element is 7		
	(iii)	If the inverse of a is a^{-1}, then		
		$a * a^{-1} = 7$	M1	
		$\Rightarrow a + a^{-1} - 7 = 7$	W1	
		$\Rightarrow a^{-1} = 14 - a$	W1	
		Therefore the inverse of a is $(14 - a)$		
	(iv)	Since $a * b = a + b - 7$, then $a * b$ is real and the set of real numbers is closed under the operation $*$.	MW1	
		Since closure and the associative law both hold, an identity exists and each element has a unique inverse then the real numbers form a group under the operation $*$	MW1	12

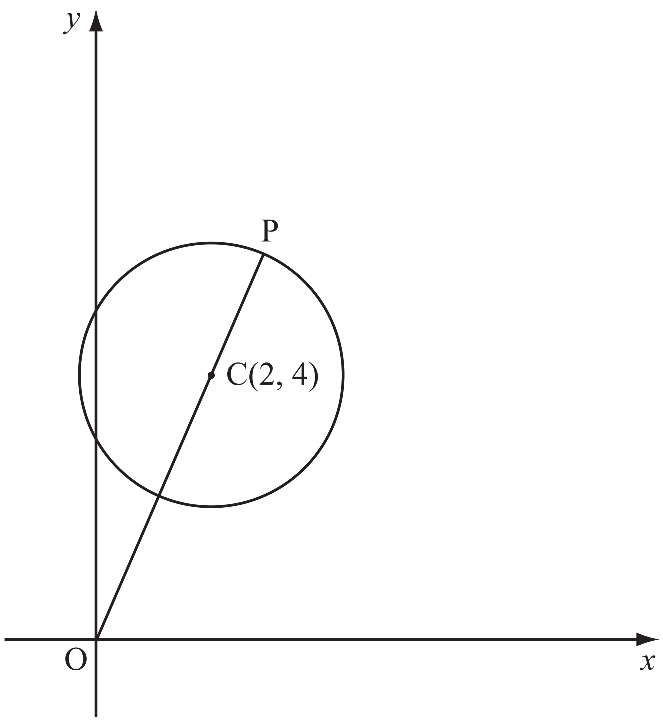
		AVAILABLE MARKS
4 (a) (i) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 & 7 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 10 & 15 \\ -3 & -7 \end{pmatrix}$		
$\Rightarrow 3a + 4b = 10$ and $3c + 4d = -3$	M1	
$7a + b = 15$ $\quad$ $7c + d = -7$	M1	
$\Rightarrow 3a + 4b = 10$ and $3c + 4d = -3$		
$28a + 4b = 60$ $\quad$ $28c + 4d = -28$	M1	
Hence $\Rightarrow 25a = 50$ and $25c = -25$		
$\Rightarrow a = 2$ $\quad$ $\Rightarrow c = -1$	W1	
$\Rightarrow b = 1$ $\quad$ $\Rightarrow d = 0$	W1	
Therefore, $\mathbf{S} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$		
(ii) Area of Q = det **S** × Area of P	M1	
But det **S** = 1	MW1	
Hence Area of Q = 1 × 12, giving an area of 12 cm^2	W1	
(b) (i) An invariant line is one where the overall line is mapped onto the same line under the transformation.	MW1	
A line of invariant points is one where each individual point on the line is mapped onto itself under the transformation.	MW1	
(ii) $\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ mx \end{pmatrix} = \begin{pmatrix} x \\ mx \end{pmatrix}$	M1 W1	
$\Rightarrow 2x + 2mx = x$	MW1	
and $x + 3mx = mx$		
Both equations give $x + 2mx = 0$	MW1	
$\Rightarrow x(1 + 2m) = 0$		
$\Rightarrow m = -\frac{1}{2}$	W1	15

5 **(i)**	Circle has centre (1, 2)	MW1	AVAILABLE MARKS
	Gradient of radius $= \frac{3-2}{3-1} = \frac{1}{2}$	MW1	
	Hence, gradient of tangent $= -2$	M1 W1	
	Therefore, the equation of the tangent is given by		
	$y - 3 = -2(x - 3)$	M1	
	$\Rightarrow y = -2x + 9$	W1	
(ii)	Substitute the point (1, 7)		
	$\Rightarrow 7 = -2 + 9$	MW1	
	$\Rightarrow 7 = 7$		
	Therefore the point (1, 7) lies on the line $y = -2x + 9$		
(iii)	Circle has centre (1, 2) and radius $\sqrt{5}$		



		MW1	
	By symmetry, the gradient of the other tangent from (1, 7) is 2.	M1 W1	
	Hence the equation of the tangent is		
	$y - 7 = 2(x - 1)$	M1	
	$\Rightarrow y = 2x + 5$	W1	12

			AVAILABLE MARKS
6 **(a)**	$(a + b\mathrm{i})^2 = -16 - 30\mathrm{i}$		
	$\Rightarrow a^2 + 2ab\mathrm{i} + b^2\mathrm{i}^2 = -16 - 30\mathrm{i}$	M1	
	Compare real components: $a^2 - b^2 = -16$	MW1	
	Compare imaginary components: $2ab = -30$	MW1	
	Hence $ab = -15$		
	$\Rightarrow b = -\dfrac{15}{a}$	W1	
	Substitute		
	$\Rightarrow a^2 - \dfrac{225}{a^2} = -16$	M1	
	$\Rightarrow a^4 + 16a^2 - 225 = 0$	W1	
	$\Rightarrow (a^2 + 25)(a^2 - 9) = 0$		
	$\Rightarrow a^2 = -25, 9$		
	$\Rightarrow a = \pm 3$	W1	
	$\Rightarrow b = \mp 5$	W1	
	Hence the solutions are: $a = 3$, $b = -5$ and $a = -3$, $b = 5$		
(b) **(i)**	Circle, centre (2, 4) and radius $\sqrt{5}$	MW3	



(ii)	Furthest point from O is P.		
	$\text{OC} = \sqrt{2^2 + 4^2} = \sqrt{20}$	M1	
	i.e. $\text{OC} = 2\sqrt{5}$	W1	
	$\text{CP} = \text{radius} = \sqrt{5}$		
	Hence maximum value of $\lvert z \rvert = 2\sqrt{5} + \sqrt{5}$	M1	
	$= 3\sqrt{5}$	W1	15
		Total	**75**