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ADVANCED
General Certificate of Education
2012

Mathematics

Assessment Unit F3

assessing

Module FP3: Further Pure Mathematics 3

[AMF31]

AMF31

THURSDAY 24 MAY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find the exact solutions of

$$8 \cosh x + 4 \sinh x = 7$$ [7]

2 Given that

$$\mathbf{a} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$$
$$\mathbf{b} = p\mathbf{i} + q\mathbf{j} + r\mathbf{k}$$

and that

$$\mathbf{a} \times \mathbf{b} = 7\mathbf{i} + \mathbf{j} + r\mathbf{k}$$

find the values of the scalar constants p, q and r. [7]

3 **(a)** Let

$$f(x) = \cos^{-1}(2x) + \cos^{-1}(-2x) \qquad -\tfrac{1}{2} < x < \tfrac{1}{2}$$

(i) Find $f'(x)$. [3]

(ii) What can be deduced from **(i)** about $f(x)$ in the interval $-\tfrac{1}{2} < x < \tfrac{1}{2}$? [1]

(iii) Evaluate $f(x)$ in the interval $-\tfrac{1}{2} < x < \tfrac{1}{2}$ [2]

(b) Given that

$$\sinh x = \tan t \qquad 0 < t < \tfrac{\pi}{2}$$

show that

$$\tanh x = \sin t$$ [4]

4 **(i)** Show that

$$\frac{\sec^2 x}{1+25\tan^2 x} \equiv \frac{1}{\cos^2 x + 25\sin^2 x}$$ [2]

(ii) Using the substitution $u = \tan x$, or otherwise, evaluate

$$\int_0^{\frac{\pi}{4}} \frac{\mathrm{d}x}{\cos^2 x + 25\sin^2 x}$$ [7]

5 **(i)** Given that $|x| < 1$ prove that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\tanh^{-1} x\right] = \frac{1}{1-x^2}$$ [4]

(ii) Using the above result show that

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$ [5]

(iii) Show that

$$\int_0^{\frac{1}{2}} \tanh^{-1} x \, \mathrm{d}x = \tfrac{3}{4}\ln 3 - \ln 2$$ [6]

6 Referred to a fixed origin O the lines L_1 and L_2 have equations

$$L_1 \quad \{\mathbf{r} - (2\mathbf{i}+4\mathbf{j}+3\mathbf{k})\} \times (2\mathbf{i}+3\mathbf{j}-\mathbf{k}) = \mathbf{0}$$
$$L_2 \quad \{\mathbf{r} - (6\mathbf{i}+3\mathbf{j}+4\mathbf{k})\} \times (\mathbf{i}-2\mathbf{j}+\mathbf{k}) = \mathbf{0}$$

(i) Show that the two lines intersect and find the position vector of the point of intersection. [7]

(ii) Find a vector that is perpendicular to both lines. [4]

(iii) Find in Cartesian form the equation of the plane containing L_1 and L_2 [3]

7 **(i)** Given that

$$I_n = \int \cosh^n x \, dx$$

show that for $n \geqslant 2$

$$nI_n = \cosh^{n-1} x \sinh x + (n-1)I_{n-2}$$ [7]

(ii) The shaded region in **Fig. 1** below is bounded by the curve $y = \cosh^3 x$, the line $x = 1$ and the x- and y-axes.

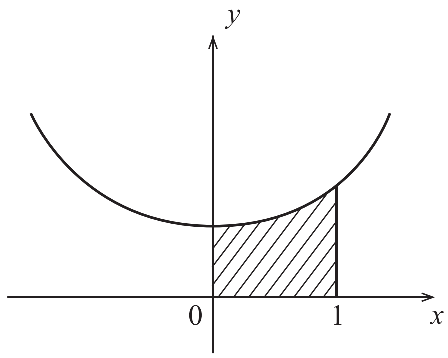

Fig. 1

Show that the area shaded is

$$\frac{e^6 + 9e^4 - 9e^2 - 1}{24e^3}$$ [6]

THIS IS THE END OF THE QUESTION PAPER