Rewarding Learning

ADVANCED
General Certificate of Education
2012

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

[AMF21]

AMF21

THURSDAY 31 MAY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Show that

$$\sum_{k=1}^{n}(k-1)k(k+1)=\frac{1}{4}(n-1)n(n+1)(n+2) \qquad [4]$$

2 Find, in radians, the general solution of the equation

$$6\sin\theta\cos\theta-2\cos\theta+3\sin\theta-1=0 \qquad [7]$$

3 **(i)** Using Maclaurin's theorem find a series expansion for ln $(1 + x)$ up to and including the term in x^5 [6]

(ii) Hence, find a series expansion for $\ln\left(\frac{1+x}{1-x}\right)$ up to and including the term in x^5 [4]

(iii) Using the expansion in part **(ii)** and substituting $x = \frac{2}{3}$, find an approximation for ln 5 [2]

4 Find, in the form pe^{iq}, the roots of the equation

$$16z^4 = 81\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$$

and plot them on an Argand diagram. [10]

5 **(i)** Use partial fractions to show that

$$\frac{5}{(2+x^2)(3+4x^2)} \equiv \frac{4}{3+4x^2} - \frac{1}{2+x^2}$$ [6]

(ii) Hence, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{y}{x} = \frac{5}{(2+x^2)(3+4x^2)}$$ [8]

6 **(i)** It is required to prove by mathematical induction that a proposition P(n) is true for all **even** natural numbers n. It has been proved that

$$\mathrm{P}(k) \Rightarrow \mathrm{P}(k+2)$$

What else must be shown to complete the proof? [2]

(ii) Prove, for all **even** natural numbers n, that

$$\frac{\mathrm{d}^n}{\mathrm{d}x^n} \sin(3x) = (-1)^{\frac{n}{2}}\, 3^n \sin(3x)$$ [9]

7 The ellipse $\frac{x^2}{29^2}+\frac{y^2}{21^2}=1$ has a general point $P(29\cos\theta, 21\sin\theta)$ and a focus F.

It is sketched in **Fig. 1** below:

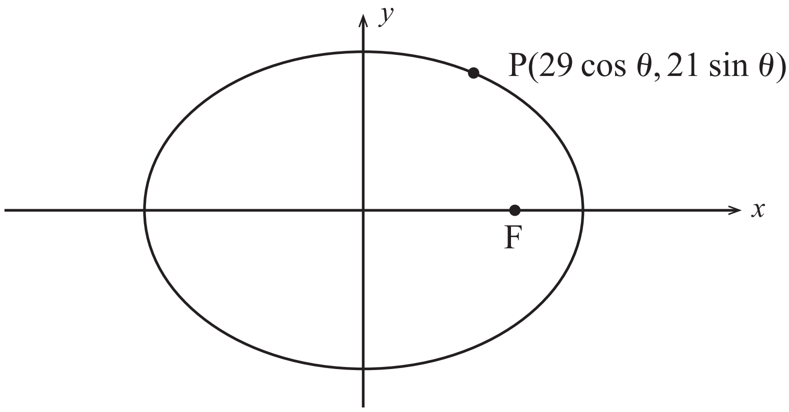

Fig. 1

(i) Show that the eccentricity of this ellipse is given by $e=\frac{20}{29}$ [2]

(ii) Show that the equation of the normal to the ellipse at P is

$$21y\cos\theta - 29x\sin\theta + 400\sin\theta\ \cos\theta = 0$$ [6]

The normal to the ellipse at P meets the x-axis at the point Q.

(iii) Show that the point Q is $\left(\frac{400}{29}\cos\theta, 0\right)$. [2]

(iv) Hence, prove that FQ $=$ eFP, where e is the eccentricity of the ellipse. [7]

THIS IS THE END OF THE QUESTION PAPER