Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2012

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

AMF11

MONDAY 25 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A system of equations is given by

$$\lambda x + 5y = 11$$
$$4x + 5\lambda y = \mu$$

(i) Find the values of λ for which the system of equations does not have a unique solution. [6]

(ii) If $\lambda = 2$, find the value of μ for which there are infinitely many solutions. [2]

2 The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 3 & 1 \\ 1 & 0 & 4 \end{pmatrix}$$

(i) Find the eigenvalues of **M**. [7]

(ii) For the eigenvalue 2, find a corresponding unit eigenvector. [5]

3 **(i)** Define clearly the symmetries of the non-square rhombus ABCD as shown in **Fig. 1** below.

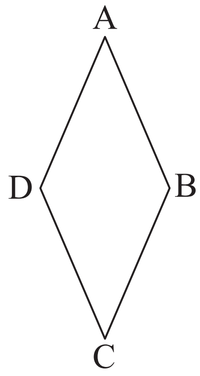

Fig. 1

[4]

(ii) Hence construct the table for the symmetry group G of this shape. [4]

The set $\{1, 4, 11, 14\}$ forms a group H under multiplication modulo 15

(iii) Draw up a table for the group H. [4]

(iv) Determine whether groups G and H are isomorphic. Justify your answer. [2]

4 **(a)** Describe fully the transformation represented by the matrix

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

[4]

(b) Find the image of the line $y = 3x - 2$ under the transformation represented by the matrix

$$\begin{pmatrix} 1 & 2 \\ 4 & -1 \end{pmatrix}$$

[4]

5 The circles C_1 and C_2 are given by the following equations.

$$C_1: \quad x^2 + y^2 + 2x - 4 = 0$$
$$C_2: \quad x^2 + y^2 + 8x + 2y - 8 = 0$$

(i) Find the points of intersection of the circles C_1 and C_2 [8]

(ii) The line $y = 2x + k$ is a tangent to the circle C_1
Find the possible values of k. [6]

6 **(a)** The complex numbers z_1 and z_2 are given as

$$z_1 = 3 + 4\text{i} \quad \text{and} \quad z_2 = 1 + p\text{i}$$

where p is a real number.

Given that the value of $z_1 + 2\,z_2$ is real, find the value of p. [3]

(b) Simplify the number

$$\frac{5-2\text{i}}{3+\text{i}}$$

giving the answer in the form $a + b\text{i}$, where a and b are real numbers. [4]

(c) **(i)** Sketch on an Argand diagram the locus of those points z which satisfy

$$|z - 3| = |z - (7 + 2\text{i})|$$

[3]

(ii) On the same diagram, sketch the locus of those points w which satisfy

$$\arg\{w - (3 + 2\text{i})\} = \frac{\pi}{4}$$

[3]

(iii) Find the point of intersection of these loci. [6]