Rewarding Learning

ADVANCED
General Certificate of Education
2012

Mathematics

Assessment Unit M3

assessing

Module M3: Mechanics 3

[AMM31]

THURSDAY 21 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A mass M of weight 140 N is suspended in equilibrium by two strings MC and MD as shown in **Fig. 1** below.

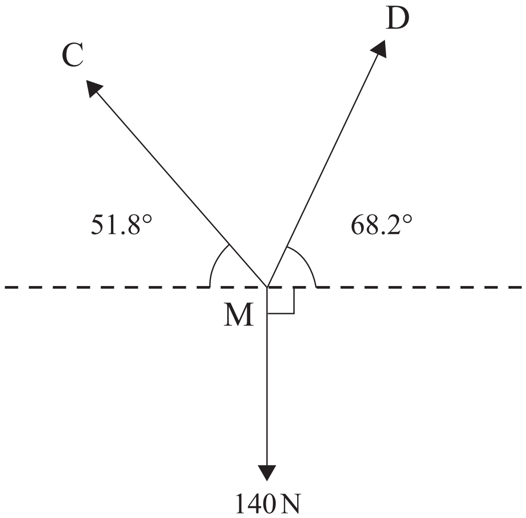

Fig. 1

The tension in MC is T_1 and the tension in MD is T_2

(i) Find T_1 and T_2 [6]

MC is an elastic string with modulus of elasticity λ.
The extension in MC is $\frac{3}{8}$ of its **extended** length.

(ii) Find λ. [4]

2 A particle P is moving along the line whose vector equation is

$$\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

under the action of two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ newtons

where $$\mathbf{F}_1 = \begin{pmatrix} 3 \\ -6 \\ -2 \end{pmatrix}$$

A and B are the two points on the line where s takes the values –1 and 1 respectively. The distance AB is measured in metres.

(i) Show that the work done by $\mathbf{F}_1$ as P is moved from A to B is 8 J. [5]

$\mathbf{R}$ is the resultant of the two forces $\mathbf{F}_1$ and $\mathbf{F}_2$

(ii) Explain why $\mathbf{R}$ is of the form

$$t \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

where t is a scalar constant. [1]

(iii) Show that the work done by $\mathbf{R}$ over the distance AB is $18t$ J. [2]

The mass of P is 0.5 kg. At A, P is moving at $5\,\text{m s}^{-1}$ and at B, $13\,\text{m s}^{-1}$

(iv) Use the Work–Energy Principle to find $\mathbf{F}_2$ [6]

3 **(i)** Using the formula for the centre of mass of a sector of a circle show that the centre of mass of a semi-circular lamina of radius r, is a distance s from its straight side where

$$s = \frac{4r}{3\pi}$$

[2]

A letter D is made from uniform plastic laminate by cutting a semi-circle of radius 90 cm from a large sheet and removing a smaller semi-circle of radius 30 cm from it so that there is 30 cm between their straight parallel sides and the letter has a horizontal axis of symmetry as shown in **Fig. 2** below.

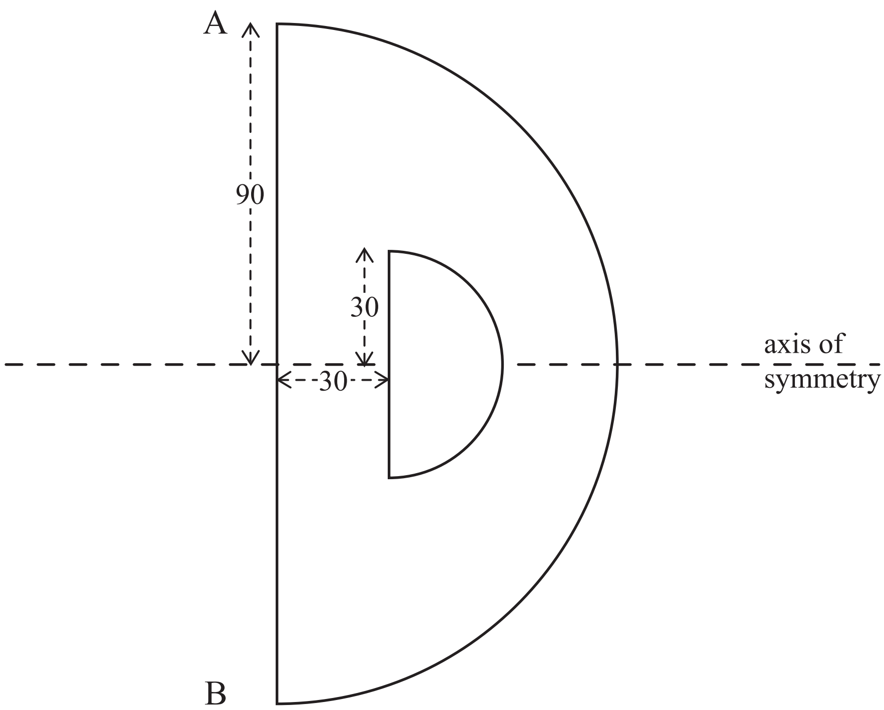

Fig. 2

(ii) If, by removing the smaller semi-circle, 1 kg of laminate is removed, show that the mass of the letter D is 8 kg. [2]

The centre of mass of the letter D is at G.

(iii) Find the distance of G from AB. [6]

The letter D hangs freely from a support at A, and is kept in equilibrium with AB vertical using the minimum possible force F.

(iv) Find F. [5]

4 A particle P is performing S.H.M. with amplitude a and period $\frac{2\pi}{\omega}$ along a straight horizontal line between the points A and B. O is the centre of the motion as shown in **Fig. 3** below.

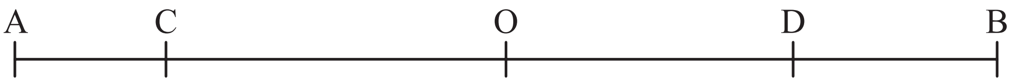

Fig. 3

Initially P is at A.
C is a point between A and O such that AC $= 0.2$ m and P's velocity at C is $1.8\,\text{ms}^{-1}$

(i) By considering the motion of P at C show that

$$\omega^2(10a - 1) = 81$$ [4]

D is a point between O and B such that OD $= 0.6$ m and P's velocity at D is $2.4\,\text{ms}^{-1}$

(ii) Find a and ω. [7]

(iii) Hence find the maximum speed and the maximum acceleration of P. [2]

5 **(i)** Show that the work done by the tension in an elastic string of natural length l and modulus of elasticity λ as its extension increases from x_1 to x_2 is

$$-\frac{\lambda}{2l}\left(x_2^{\ 2}-x_1^{\ 2}\right)$$ [3]

A particle of mass m is attached to two elastic strings PA and PB each of natural length $2a$ and modulus of elasticity $0.5\,m$g. The ends A and B are attached to two fixed points on the same horizontal beam. The particle is held at rest with APB horizontal and PA = PB = $4a$. The particle is released from rest and passes through a point Q with speed v where Q is a distance $3a$ vertically below the particle's starting point as shown in **Fig. 4** below.

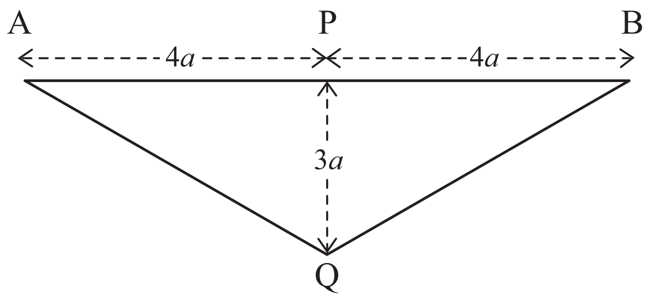

Fig. 4

(ii) Show that the total work done by the forces acting on the particle in moving it from P to Q is $1.75\,m$ga. [4]

(iii) Find v in terms of g and a. [2]

6 Search and rescue planners are investigating how prevailing wind speed and direction affect the time for search flights.
They are considering the effect on an aircraft travelling from its base B to a point P which is due south of B.
The wind is blowing at u kmh^{-1} from a direction bearing $(180° + \theta)$.
The aircraft flies at V kmh^{-1} relative to the wind.
The aircraft is set on a course bearing $(180° + \alpha)$ as shown in **Fig. 5** below.

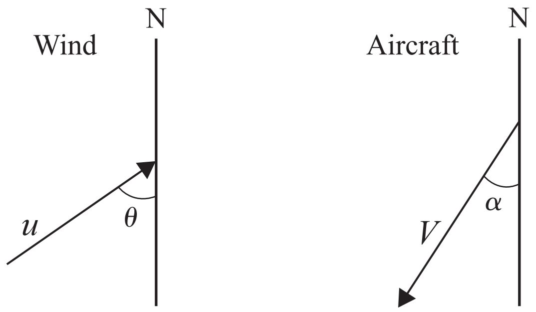

Fig. 5

(i) Using a velocity diagram, or otherwise, show that $u \sin \theta = V \sin \alpha$. [2]

For the return flight from P back to B the aircraft is set on a course bearing $(360° - \beta)$.

(ii) Find $\sin \beta$ in terms of u, V and θ and hence show that $\beta = \alpha$. [3]

The aircraft takes T hours for the flight from B to P and then back to B.
The distance from B to P is d kilometres.

(iii) Show that $T = \dfrac{2dV\cos\alpha}{V^2\cos^2\alpha - u^2\cos^2\theta}$ [6]

(iv) Hence show that $T = \dfrac{2d\sqrt{V^2 - u^2\sin^2\theta}}{V^2 - u^2}$ [2]

(v) What is the advantage in having an expression for T that is independent of α? [1]

THIS IS THE END OF THE QUESTION PAPER