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ADVANCED
General Certificate of Education
2012

Mathematics

Assessment Unit M4

assessing

Module M4: Mechanics 4

[AMM41]

FRIDAY 22 JUNE, AFTERNOON

AMM41

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(a)** ABC is a framework of three light pin jointed rods freely hinged to a rigid support at B. The framework supports a weight of 30 N at C. It is held in equilibrium with BC horizontal by a force P acting at A in the direction CA as shown in **Fig. 1** below.

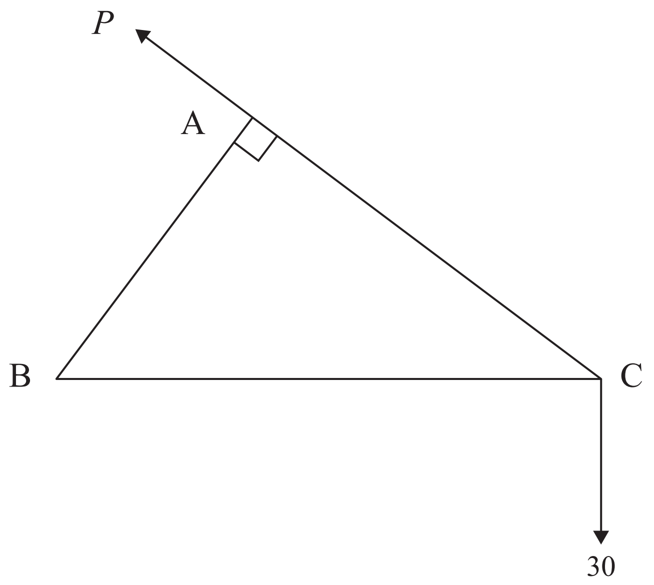

Fig. 1

AB = 0.3 m   AC = 0.4 m   BC = 0.5 m

(i) Find P. [3]

(ii) Explain why there is no force in AB. [1]

(iii) Find the internal forces in the rods AC and BC. [3]

(b) **Fig. 2** below shows a scalene quadrilateral ABCD with
AB = 0.9 m   BC = 1.2 m   CD = 0.8 m   DA = 1.7 m   AC = 1.5 m

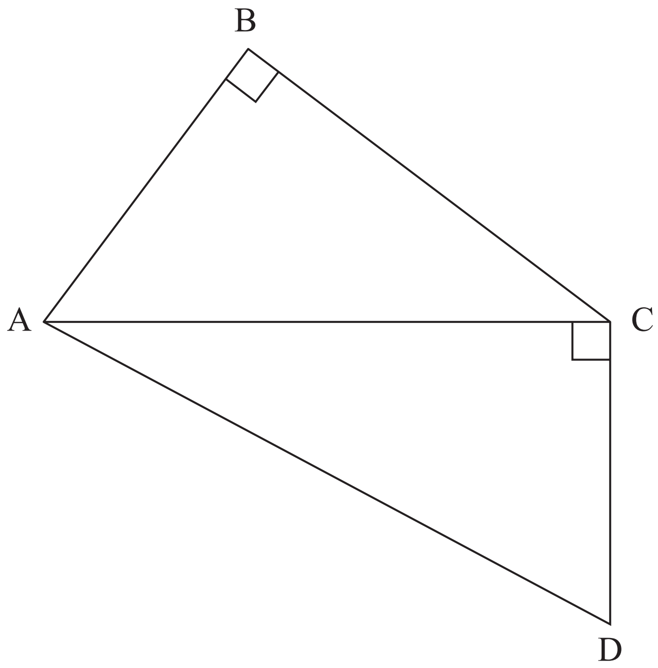

Fig. 2

Forces of 9, 12, 8 and 17 N act along the sides AB, BC, CD and DA respectively.

Show that this system reduces to a couple and find its moment. [7]

2 A hovercraft is kept at its equilibrium height above the ground by a flow of gas of density d and cross-sectional area A thrusting downwards with velocity v. Experimental trials have shown that the total weight W, that the down-thrust can support, is given by

$$W = cAdv^2$$

where c is a dimensionless constant.

(i) Show that this formula is dimensionally consistent. [3]

The power developed for vertical thrust by the engine is P.

(ii) Show that $[P] = [\mathrm{M}][\mathrm{L}]^2[\mathrm{T}]^{-3}$ [2]

The power is believed to depend on the properties of the gas jets as follows:

$$P = kA^x d^y v^z$$

where k is a dimensionless constant.

(iii) Use the Method of Dimensions to find x, y and z. [6]

(iv) Hence confirm that the power to weight ratio of the hovercraft is proportional to v. [1]

3 Three particles, A, B and C with masses $2m$, m and km respectively lie in a straight line on a smooth horizontal surface. B and C are at rest and A is moving towards B with speed u. A collides directly with B. After the collision B moves off with speed v_2 and A follows at speed v_1. The coefficient of restitution between any pair of particles is 0.5

(i) Show that $v_2 = u$ and find v_1 [6]

B then collides directly with C. C moves off with speed w_2 and B follows at speed w_1

(ii) Find w_2 and show that w_1 is given by

$$w_1 = w_2 - \frac{u}{2}$$ [4]

(iii) Given that there will be at least three collisions, show that $k > \frac{1}{2}$ [2]

4 A particle of mass m is moving round a vertical circle of radius r and vertical diameter AOB where O is the centre of the circle and A is above B. When the particle is at P its speed is $v\,\text{ms}^{-1}$ and the angle AOP is θ as shown in **Fig. 3** below.

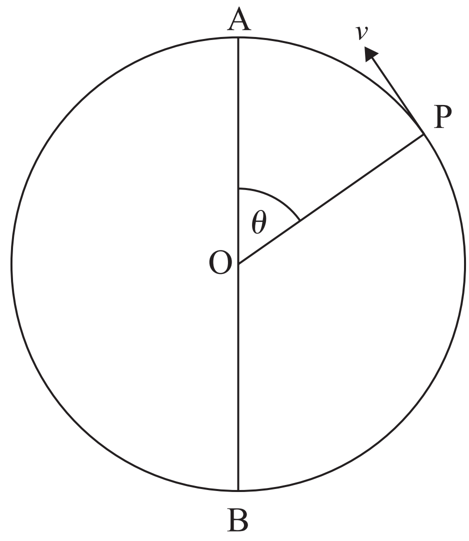

Fig. 3

If the gravitational potential energy is zero at B, the kinetic energy of the particle at P is

$$mgr(1 - \cos\theta)$$

(i) Find the initial speed of the particle if it was projected from B. [2]

(ii) If the particle is a bead free to move round a vertical circle formed by a smooth fixed rigid wire, find in terms of m, g and θ, an expression for the reaction of the wire on the bead at P. [4]

(iii) If instead, the particle is attached to the end of a light inextensible string and moves in an arc of a vertical circle, find θ when the tension in the string vanishes. [2]

(iv) Show that the particle in **(ii)** can make complete circles but that the particle in **(iii)** cannot. [3]

5 A car of mass m kg is travelling at $v \text{ ms}^{-1}$ in a horizontal circle of radius 50 m round a bend banked at 45° to the horizontal as shown in **Fig. 4** below.

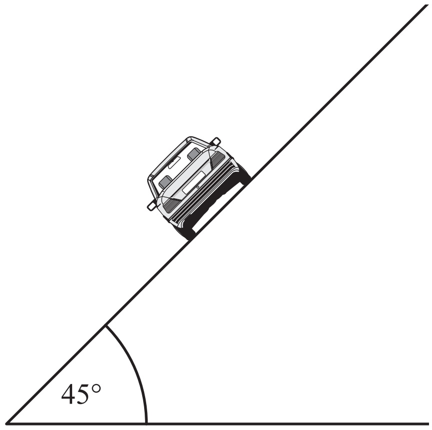

Fig. 4

The coefficient of friction between the car and the road surface is μ.

(i) If the car is just about to slip up the slope, show that

$$v^2 = \frac{490(1+\mu)}{(1-\mu)}$$

[8]

When the car is travelling more slowly at $u \text{ ms}^{-1}$ it is just about to slip down the slope.

(ii) Hence **write down** in terms of μ an expression for u^2, clearly explaining why this can be done. [2]

(iii) If $\mu = 0.5$, show that $v = 3u$. [2]

6 A paperweight can be modelled by the solid formed when the area bounded by the positive x-axis, the y-axis and the curve

$$y = \sqrt{1 - \frac{x^2}{4}}$$

is rotated through 2π radians about the x-axis as shown in **Fig. 5** below.

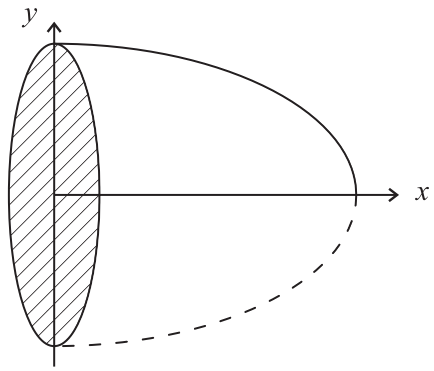

Fig. 5

The density of the solid formed is ρ.

(i) Show that the moment of this solid about the y-axis is $\pi\rho$. [5]

(ii) Hence find the distance of the centre of mass of this solid from its plane face. [4]

The paperweight is placed with its plane face in contact with the horizontal surface of a tilting table. When the table is tilted through $\alpha°$, the solid is just on the point of toppling but does not slip.

(iii) Find α. [5]

THIS IS THE END OF THE QUESTION PAPER

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