Rewarding Learning

ADVANCED
General Certificate of Education
2012

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

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FRIDAY 1 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(a)** For each relation below state if it is a one–one function, a many–one function or a mapping.

(i) $\text{a}: x \to \pm\sqrt{x} \qquad x \in \mathbb{R} \quad x \geqslant 0$ [1]

(ii) $\text{b}: x \to \dfrac{1}{x} \qquad x \in \mathbb{R} \quad x \neq 0$ [1]

(iii) $\text{c}: x \to x^2 \qquad x \in \mathbb{R}$ [1]

(b) The function f is defined by

$$\text{f}(x) = x^2 - 3 \qquad x \in \mathbb{R}$$

and the function g is defined by

$$\text{g}(x) = 2x + 1 \qquad x \in \mathbb{R}$$

(i) Find the composite function $\text{gf}(x)$, stating its domain. [3]

(ii) Find the values of x such that $\text{gf}(x) = 3x$ [3]

2 **(i)** Sketch the graph of

$$y = 4 - x^2$$

clearly showing where it crosses the x-axis. [2]

A paperweight can be modelled as the solid formed when the area bounded by the curve

$$y = 4 - x^2$$

and the x-axis and the y-axis is rotated through 360° about the x-axis.

(ii) Find the volume of the paperweight. [7]

3 During a science experiment, students create two waves in a ripple tank.
The first wave can be modelled by the equation

$$h_1 = 6\cos t \qquad 0 \leqslant t \leqslant 2\pi$$

The second wave can be modelled by the equation

$$h_2 = 5\sin t \qquad 0 \leqslant t \leqslant 2\pi$$

where h_1 and h_2 are the heights of the waves, measured in millimetres, at any time t seconds from the start of the experiment.
The two waves join together.

(i) Express the height $h_1 + h_2$ of the resultant wave in the form

$$r\cos(t - \alpha) \qquad 0 \leqslant \alpha \leqslant \frac{\pi}{2} \qquad r \in \mathbb{R}$$ [4]

(ii) Hence find the times when the resultant wave has a height of 3 mm. [4]

4 **(i)** Given that

$$x^2 + 6xy + y^2 + 32 = 0$$

use implicit differentiation to show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-(x+3y)}{3x+y}$$

[5]

(ii) Hence find the coordinates of the stationary points on the curve

$$x^2 + 6xy + y^2 + 32 = 0$$ [6]

5 Solve the differential equation

$$\cos^2 4x \frac{\mathrm{d}y}{\mathrm{d}x} = y$$

given that $y = \mathrm{e}^3$ when $x = \frac{\pi}{16}$ [8]

6 **(i)** Find the vector equation of the line l through the points (2, 4, 3) and (1, 2, 3). [4]

The point P lies on the line l.
O is the point (0, 0, 0).

(ii) Hence, using **(i)**, write down the position vector $\overrightarrow{OP}$ [1]

OP is perpendicular to the line l.

(iii) Find the coordinates of the point P. [6]

7 Find the **exact** value of

$$\int_1^3 \ln x \, dx$$ [7]

8 **(i)** Prove the identity

$$\tan 3\theta \equiv \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$ [7]

(ii) Hence solve the equation

$$\tan 3\theta = \tan\theta \qquad 0° \leqslant \theta \leqslant 360°$$ [5]

THIS IS THE END OF THE QUESTION PAPER