Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2013

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

AMF11

FRIDAY 11 JANUARY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Two circles have equations

$$x^2 + y^2 - 2x - 4y - 20 = 0$$

$$x^2 + y^2 - 10x + 20 = 0$$

Find the points where these circles intersect. [8]

2 The matrix **M** has eigenvalues 1 and 3 with corresponding eigenvectors $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

(i) Find **M** [6]

U is a 2 × 2 matrix such that $\mathbf{U}^{-1}\,\mathbf{M}\,\mathbf{U} = \mathbf{D}$, where **D** is a diagonal matrix.

(ii) Write down a possible matrix **D** [1]

3 Let $\mathbf{N} = \begin{pmatrix} p & 2 & 4 \\ 1 & 0 & -5 \\ 0 & 3 & 1 \end{pmatrix}$

(i) Find the rational value of p for which this matrix does not have an inverse. [4]

(ii) If $p = 1$, find the inverse of **N** [7]

(iii) Hence solve the following system of equations

$$x + 2y + 4z = 5$$

$$x - 5z = 3$$

$$3y + z = 3$$ [4]

4 **(a)** Describe fully the transformation represented by the matrix $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ [3]

(b) The matrix $\mathbf{A} = \begin{pmatrix} -1 & 3 \\ 2 & 0 \end{pmatrix}$

The point P is mapped to Q (13, 4) by the transformation represented by **A**

Find the coordinates of P. [4]

(c) The matrix $\mathbf{B} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$ represents a linear transformation of the x-y plane.

The line $y = mx$ is rotated through 90° about the origin under the transformation represented by **B**

Find the possible values of m. [8]

5 The group G consists of the elements $\{e, a, a^2, b, ab, a^2b\}$ under multiplication.

e is the identity, $a^3 = b^2 = e$ and $ab = ba$

(i) Show that $bab = a$ [2]

(ii) Hence simplify as far as possible the product $abab$ [2]

(iii) Hence show that the product $a^2ba^2b = a$ [4]

(iv) State which elements of G are self-inverse. [2]

The multiplication table for a group H is shown below.

	i	p	q	r	s	t
i	i	p	q	r	s	t
p	p	q	i	t	r	s
q	q	i	p	s	t	r
r	r	s	t	i	p	q
s	s	t	r	q	i	p
t	t	r	s	p	q	i

(v) Are the groups G and H isomorphic? Explain your answer carefully. [2]

6 **(a)** Find the complex roots of the equation

$$5z^2 + 8z + 5 = 0$$ [4]

(b) The complex numbers p and q are given by

$$p = 5 + 2a\mathrm{i}, \qquad q = 7 - a\mathrm{i}$$

where a is a real number.

Given that $|p| = |q|$, find the exact values of a. [5]

(c) **(i)** Sketch on an Argand diagram the locus of those points z which satisfy

$$|z - (6 + 4\mathrm{i})| = 3$$ [3]

(ii) On the same Argand diagram sketch the locus of those points w which satisfy

$$\arg\{w - (1 - 2\mathrm{i})\} = \frac{\pi}{2}$$ [3]

(iii) Find the minimum value of $|z - w|$, where z and w are complex numbers, satisfying the equations in **(i)** and **(ii)** respectively.

A solution by scale drawing will not be accepted. [3]

THIS IS THE END OF THE QUESTION PAPER