Rewarding Learning

ADVANCED
General Certificate of Education
January 2013

Mathematics

Assessment Unit M2

assessing

Module M2: Mechanics 2

[AMM21]

AMM21

THURSDAY 31 JANUARY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 At time $t = 0$ seconds, a particle P is projected from a fixed point O.
P moves so that, at any time t, $t \geqslant 0$, its position vector $\mathbf{r}$ metres relative to O is given by

$$\mathbf{r} = (t^3 - 2t^2)\mathbf{i} + (4t^2 - t)\mathbf{j}$$

(i) Find the **speed** of P when $t = 2$ [5]

(ii) Find the acceleration of P when $t = 5$ [3]

2 A pump draws petrol from an underground storage tank at the rate of $0.24\,\text{m}^3\,\text{min}^{-1}$ and issues it at a speed of $5\,\text{m}\,\text{s}^{-1}$ from a nozzle which is 5 m vertically above the level from which the petrol is drawn.
Take the density of the petrol to be $740\,\text{kg}\,\text{m}^{-3}$

(i) Show that the mass of the petrol issued from the pipe in 1 second is 2.96 kg. [2]

(ii) Find the power developed by the pump. [5]

3 An aeroplane of mass m kg is travelling at a constant speed of $v\ \text{m s}^{-1}$ in a horizontal circular path of radius r metres.
The aeroplane banks at an angle of θ to the vertical which creates a lift force L newtons which acts perpendicular to the wings as shown in **Fig. 1** below.

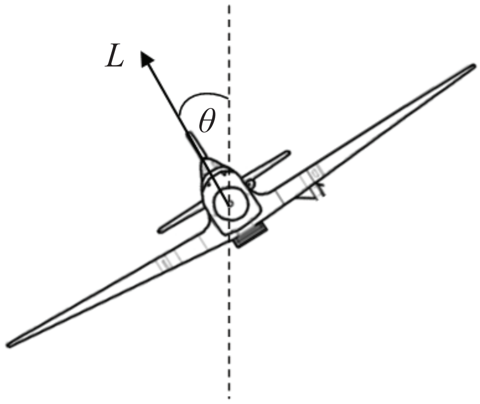

Fig. 1

(i) Show that

$$\tan\theta = \frac{v^2}{rg}$$ [6]

The plane has a mass of 2×10^5 kg and is travelling at a constant speed of $200\ \text{m s}^{-1}$ in a horizontal circular path of radius 15 000 metres.

(ii) Find L. [4]

4 **Fig. 2** below shows a skateboarder, John, of mass 70 kg, setting off with an initial speed of $2\,\text{m}\,\text{s}^{-1}$ **down** the line of greatest slope of a ramp.
The ramp is 80 m long and is inclined at a constant 20° to the horizontal.

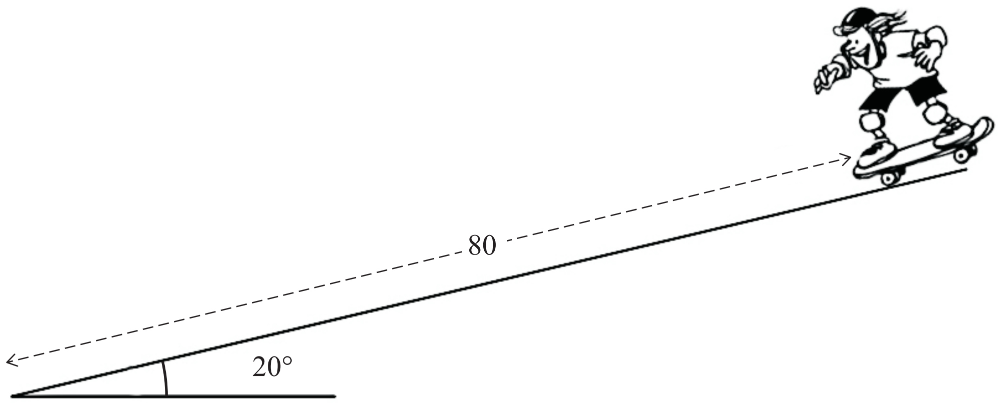

Fig. 2

Take John's potential energy to be zero at the bottom of the ramp.

(i) Find the potential energy John loses while skating from the top to the bottom of the ramp. [3]

(ii) Assuming that resistances total zero, calculate John's speed at the bottom of the ramp. [5]

He actually reaches the bottom of the ramp with a speed of $12\,\text{m}\,\text{s}^{-1}$, so a resistive force must have been acting along the ramp.

(iii) Calculate the magnitude of this resistive force, which is assumed to be constant. [4]

5 A woman of mass 63 kg runs along a horizontal track at a constant speed of $4\,\mathrm{m\,s^{-1}}$
In order to overcome resistance, she works at a constant rate of 120 W.

(i) Show that the magnitude of the resistance she experiences is 30 N. [3]

She now comes to a hill inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{15}$
To allow for the hill, she reduces her speed to $3\,\mathrm{m\,s^{-1}}$ and maintains this constant speed as she runs up the hill.
Assume that the resistance she experiences remains at 30 N.

(ii) Find the rate at which the woman has to work to run up the hill. [4]

When she reaches the top of the hill, the ground once again becomes horizontal.
She increases her speed to $5\,\mathrm{m\,s^{-1}}$ and works at a constant rate of 250 W.
The resistance remains at 30 N.

(iii) Find her acceleration. [5]

6 **[In this question take $\mathbf{g = 10\,m\,s^{-2}}$]**

A particle P of mass m kg is projected **vertically upwards** with velocity $u\,\mathrm{m\,s^{-1}}$ from a point O on horizontal ground.
P is subject to a resistance of magnitude $5mv$ N, where v is the velocity of the particle when it is x metres above O.

(i) Show that the equation of motion of P can be modelled by

$$v\frac{\mathrm{d}v}{\mathrm{d}x} = -5(2+v)$$ [4]

(ii) Find x in terms of u and v. [9]

(iii) Given that $u = 10\,\mathrm{m\,s^{-1}}$, find the greatest height of P above O. [2]

7 A hockey player scoops a ball towards the goal net.
He scoops the ball at u m s^{-1} at an angle of 30° to the horizontal.
The goal net is 1.5 metres high and the ball is initially 8 metres directly in front of the base of the net, as shown in **Fig. 3** below.

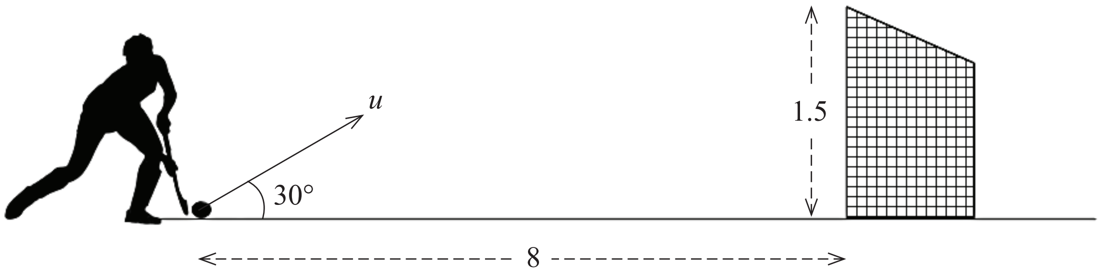

Fig. 3

Assume all motion occurs in the same vertical plane.

(i) Find the maximum value of u for which the ball will travel below the crossbar and score a goal. [6]

(ii) Taking $u = 7.35$ m s^{-1}, find the **speed** of the ball 1 second into its flight time. [5]

THIS IS THE END OF THE QUESTION PAPER