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ADVANCED
General Certificate of Education
2013

Mathematics

Assessment Unit C3
assessing
Module C3: Core Mathematics 3

[AMC31]

AMC31

FRIDAY 17 MAY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that
$\ln z \equiv \log_e z$

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Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Differentiate:

(i) $x\cos x$ [3]

(ii) $\dfrac{\tan 2x}{x}$ [5]

(iii) $\ln(x^2 + 3)$ [2]

2 **(a)** Simplify as far as possible

$$\frac{(2x^2 + 7x - 15)(2x - 14)}{(x^2 - 2x - 35)(4x^2 - 9)}$$ [5]

(b) Solve the inequality

$$|2x - 3| < 1$$ [5]

3 **(i)** Find the first 3 terms in the binomial expansion of

$$(1 - 2x)^{\frac{1}{3}}$$ [4]

(ii) **Hence**, clearly showing your method, find an approximation to $\sqrt[3]{0.88}$ without using a calculator. [2]

4 The graph of a function $y = \mathrm{f}(x)$ is sketched below in **Fig. 1**

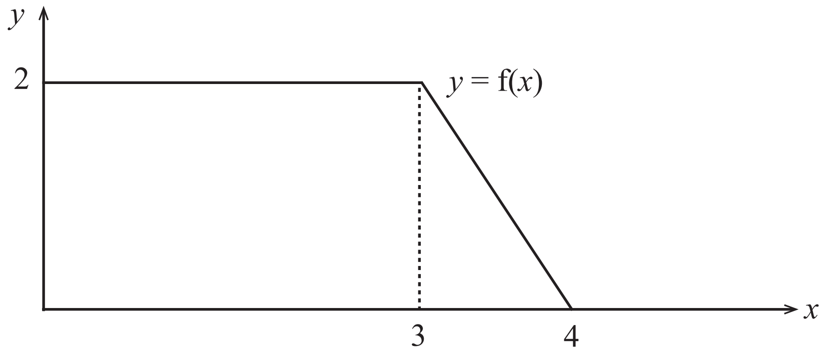

Fig. 1

On separate diagrams sketch the graphs of:

(i) $y = 1 - \mathrm{f}(x)$ [3]

(ii) $y = \frac{1}{2}\mathrm{f}(x+2)$ [3]

labelling clearly where the graphs cross or touch the axes.

5 **(a)** The temperature, C, of an ingot of cooling metal can be modelled by

$$C = 12 + 80\,\mathrm{e}^{\frac{-t}{30}}$$

where t is measured in minutes.

Find C when $t = 20$ [2]

(b) Find a Cartesian equation of the curve defined parametrically by

$$x = \mathrm{e}^{-t} \qquad y = 3 + \mathrm{e}^{2t}$$ [4]

6 **(a)** **(i)** Use partial fractions to rewrite

$$\frac{x+10}{(2x-5)x^2} \quad \text{in the form} \quad \frac{A}{2x-5}+\frac{B}{x}+\frac{C}{x^2}$$

where A, B and C are integers. [6]

(ii) Hence find

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x+10}{(2x-5)x^2}\right)$$ [3]

(b) Solve

$$\cot 2\theta = -\frac{4}{3}$$

where $0 \leqslant \theta \leqslant 2\pi$ [4]

7 **Fig. 2** below shows the graph of $y = x^2 + 1.3$ and $y = 2 \cos x$.

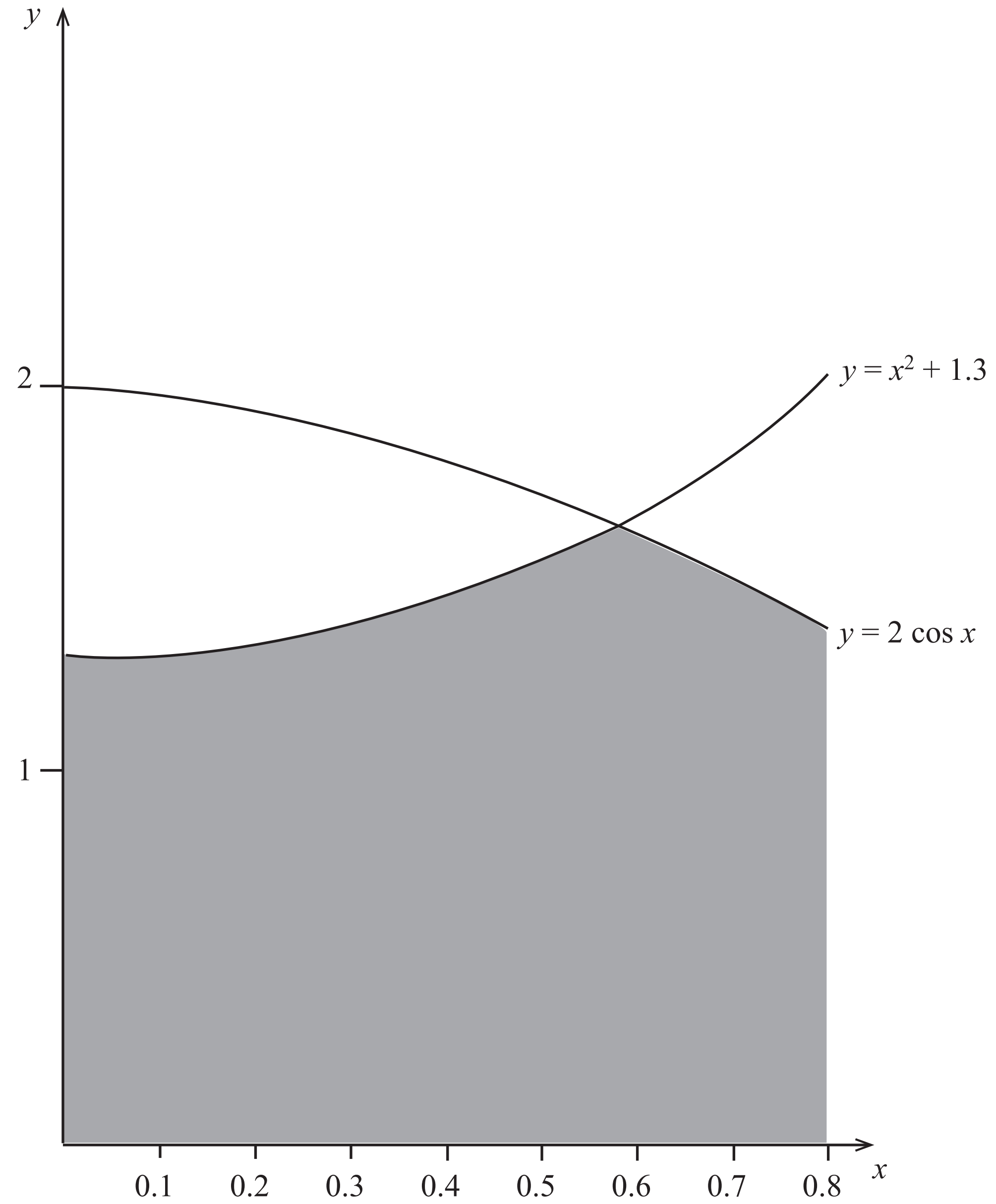

Fig. 2

(i) Taking $x_0 = 0.5$ as a first approximation, use the Newton-Raphson method twice to find a better approximation to the x-coordinate of the point of intersection of the curves. [8]

A glazier designs a pane of a stained glass window identical to the shaded area in **Fig. 2** above, i.e. the area between the curves, the axes and the line $x = 0.8$

(ii) Use Simpson's rule with 5 ordinates to find an approximation to the area of the pane. [6]

8 **(i)** Prove the identity

$$\tan A \sec A + \frac{1}{1+\sin A} \equiv \sec^2 A$$ [5]

(ii) Hence find

$$\int\left(\tan 2x \sec 2x - \frac{3}{x} + \mathrm{e}^{2x} + \frac{1}{1+\sin 2x}\right)\mathrm{d}x$$ [5]

THIS IS THE END OF THE QUESTION PAPER