Rewarding Learning

ADVANCED
General Certificate of Education
2013

Mathematics

Assessment Unit M2

assessing

Module M2: Mechanics 2

[AMM21]

AMM21

THURSDAY 13 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A particle of mass 4 kg is acted upon by a force, **P**, so that its position vector **r** relative to a fixed point O, at time t seconds is given by

$$\mathbf{r} = (4t^2 - 5t)\mathbf{i} + (16 - t^2)\mathbf{j} \text{ metres}$$

(i) Find an expression for the velocity of the particle at any time t. [2]

(ii) Show that the acceleration of the particle is constant. [3]

(iii) Find **P**. [2]

2 A ball of mass 0.5 kg is travelling across a smooth horizontal surface. The ball is then struck by a bat.
Immediately before being struck the ball had a velocity of $(-2\mathbf{i} + 4\mathbf{j})$ m s^{-1} and immediately after being struck the velocity of the ball is $(5\mathbf{i} - 10\mathbf{j})$ m s^{-1}

(i) Find the magnitude of the impulse exerted on the ball by the bat. [4]

At the moment of being struck the position vector of the ball was $(-2\mathbf{i} - 2\mathbf{j})$ m.

(ii) Find the position vector of the ball after 3 seconds. [4]

3 The mass of a car is 800 kg.
The engine of the car works at a constant rate of 20 kW, and the motion of the car is subject to a constant resistance of magnitude 600 N.

(i) Find the acceleration of the car when it is travelling at 25 m s^{-1} on a level road. [4]

The car now ascends a straight road, inclined at 5° to the horizontal, with the same power output and against the same constant resistance.

(ii) Find the maximum speed at which the car can ascend the road. [4]

4 A postman places a parcel of mass 4 kg at the bottom of a rough ramp which is inclined at 30° to the horizontal as shown in **Fig. 1** below.

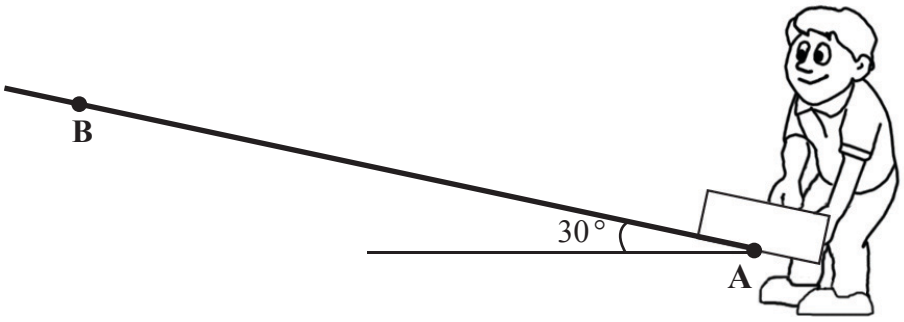

Fig. 1

The coefficient of friction between the ramp and the parcel is 0.7

The postman pushes the parcel up the ramp giving it an initial speed of 5 ms^{-1} at point A.
The parcel stops at point B.
The distance AB = d metres.

Model the parcel as a particle.

Take the potential energy at A to be zero.

(i) Find the kinetic energy of the parcel at A. [2]

(ii) Find, in terms of d, the work done against gravity in moving the parcel from A to B. [3]

(iii) Find, in terms of d, the work done against friction in moving the parcel from A to B. [4]

(iv) Hence, find d. [2]

5 **Fig. 2** below illustrates a bullet of mass 0.1 kg being fired from a gun directly at a block of wood.

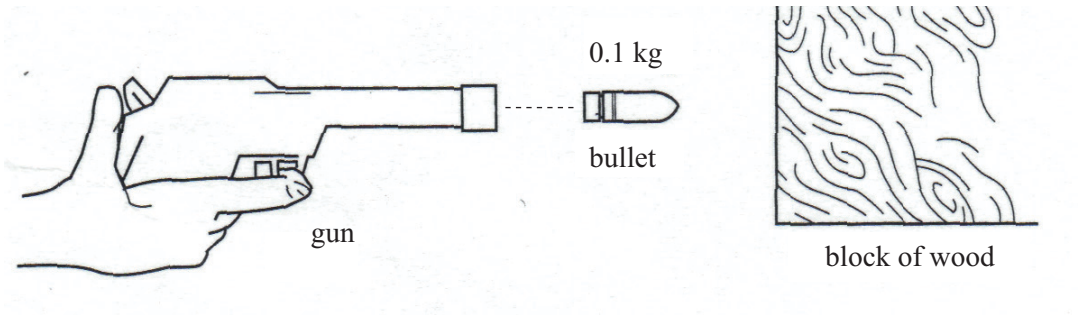

Fig. 2

The bullet leaves the gun and hits the block of wood with an initial speed of 100 $\mathrm{m\,s^{-1}}$. When travelling through the block of wood, the bullet is subject to a resistive force of $20v^2$ newtons, where v $\mathrm{m\,s^{-1}}$ is the velocity of the bullet when it has penetrated a distance x metres into the wood.

(i) Show that the equation of the motion of the bullet through the wood can be modelled by

$$-200v = \frac{\mathrm{d}v}{\mathrm{d}x}$$ [3]

(ii) Find x when the speed of the bullet has been reduced to 25 $\mathrm{m\,s^{-1}}$ [8]

(iii) Explain briefly why this model could **not** be used to find the distance travelled by the bullet through the block of wood before the bullet comes to rest.
[Assume the bullet does not exit the block.] [2]

6 A glass marble of mass m kg is moving in a horizontal circle around the inside surface of a smooth hemispherical bowl of diameter $4x$ metres.
The centre of the circular path of the marble is at a distance of x metres below the centre, C, of the bowl, as shown in **Fig. 3** below.

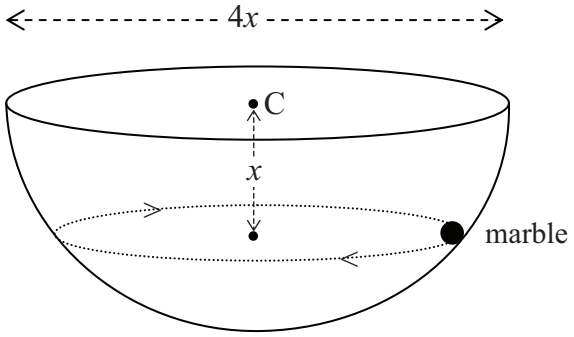

Fig. 3

(i) Draw a diagram showing the external forces acting on the marble. [2]

(ii) Show that the marble travels in a circle of radius $\sqrt{3}\,x$ metres. [1]

(iii) Find, in terms of m and g, the magnitude of the reaction between the marble and the bowl. [4]

(iv) Find, in terms of x and g, an expression for the speed of the marble. [5]

7 **[In this question take g = 10 m s^{-2}]**

A particle is projected from a point O at an angle of θ **below** the horizontal with a speed of u m s^{-1}
The position P of the particle when it has travelled x metres horizontally and dropped a distance of y metres vertically below the level of O is shown in **Fig. 4** below.

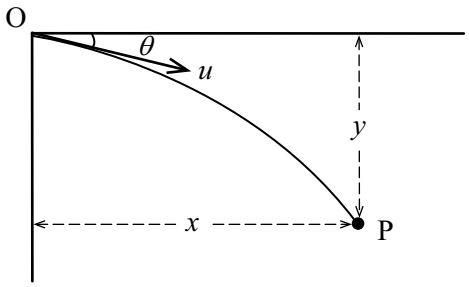

Fig. 4

(i) Prove that the equation of the path of **this** projectile is given by

$$y = x \tan \theta + \frac{gx^2}{2u^2} (1 + \tan^2 \theta)$$ [8]

A ball is projected with an initial speed of 18 $m s^{-1}$ at an angle of 25° below the horizontal from a point on the top of a vertical wall. The point of projection is 10 m vertically above horizontal ground. The ball hits a vertical fence which is at a horizontal distance of 12 m from the wall as shown in **Fig. 5** below.

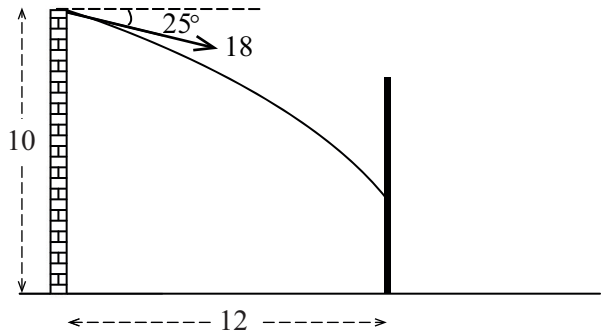

Fig. 5

(ii) Calculate the height, above the ground, of the point at which the ball hits the fence. [3]

The fence is 6 m high. A ball is now projected from the same point with the same initial speed of 18 $m s^{-1}$ at an angle of θ below the horizontal, so that it just clears the fence.

(iii) Find the minimum value of θ. [5]

THIS IS THE END OF THE QUESTION PAPER