Rewarding Learning

ADVANCED
General Certificate of Education
2013

Mathematics

Assessment Unit M3

assessing

Module M3: Mechanics 3

AMM31

[AMM31]

TUESDAY 18 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **Fig. 1** below shows a box of weight 60 N suspended from a rigid fixed horizontal beam by two vertical elastic strings S_1 and S_2

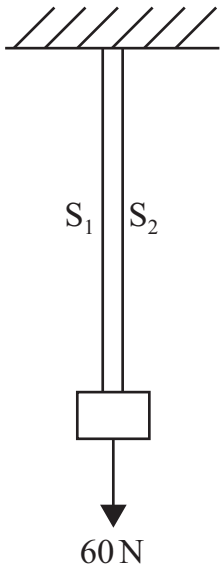

Fig. 1

S_1 is of natural length $4l$ and modulus λ.
S_2 is of natural length $5l$ and modulus $\frac{1}{3}\lambda$.
When the box is in equilibrium, S_1 is twice its natural length.
Model the box as a particle.

(i) Find, in terms of l, the extension in S_2 [2]

(ii) Find λ. [6]

2 **Fig. 2** below shows a quadrant of a circle of radius 90 cm.
The centre of mass of this quadrant is at a distance d cm from each straight side.

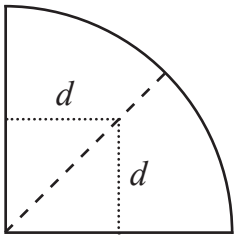

Fig. 2

(i) Using the formula for the position of the centre of mass of a sector of a circle show that $d = \frac{120}{\pi}$ [4]

Four congruent laminae each in the shape of a quadrant of a circle of radius 90 cm are joined together to form a tulip shaped logo for a garden centre as shown in **Fig. 3** below.

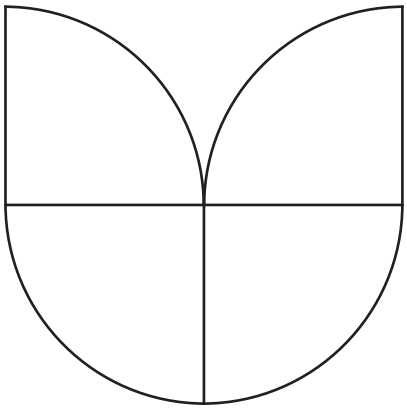

Fig. 3

(ii) Show that the centre of mass is at the point G where all four quadrants meet. [4]

A new logo is constructed where the two upper laminae each have density σ and the two lower laminae each have density 2σ.

(iii) Find the distance of the centre of mass of this new logo below G. [5]

3 In this question take **i** and **j** to be unit vectors with directions due East and due North respectively.
On the first leg of a course a yacht is travelling due North at 12 km h^{-1}
To the crew the wind appears to be blowing from the West at u km h^{-1}
The actual velocity of the wind is **v** km h^{-1}

(i) Find **v** in terms of u, **i** and **j**. [3]

On the second leg of the course the yacht is travelling Nθ° E at 15 km h^{-1}, where $\tan \theta° = \frac{4}{3}$

The wind now appears to be blowing from the South at w km h^{-1}

(ii) Find **v** in terms of w, **i** and **j**. [3]

(iii) Find the actual speed of the wind and the direction from which it is blowing. [4]

(iv) Find on which leg of the course the wind **appears** stronger. [3]

4 In Extreme Bungee Jumping participants descend vertically holding weights and release them at the bottom of the descent.
Vlad, whose mass is 50 kg, has been assured that there is ample clearance below. Holding two 5 kg sandbags he drops from a high platform. He is attached to a bungee cord of natural length 15 m and modulus 300 g N. The other end of the cord is attached to the platform.
He comes to instantaneous rest at S, where the extension in the cord is d metres.
Take S to be the zero level for gravitational potential energy.

(i) Show that the exact value of d is $3 + 3\sqrt{11}$ [7]

At S Vlad drops the sandbags, rebounds upwards and instantaneously comes to rest at a height of h metres vertically above S, with the cord now slack.

(ii) Show that $h = \dfrac{6(d+15)}{5}$. [4]

(iii) By considering the result in **(ii)** explain briefly why this form of bungee jumping could be more dangerous to the participant than the normal version. [1]

5 At $t = 0$ seconds a light spot on a screen is observed moving horizontally with S.H.M. It has displacement x_1 metres from a fixed point O where

$$x_1 = 4 \sin\left(\frac{\pi}{2}t + \varepsilon\right)$$

(i) Write down the amplitude and period of this motion. [2]

(ii) Under what initial condition is $\varepsilon = 0$? [1]

When $t = 0$ and $x_1 = 2\sqrt{2}$ the spot is moving towards the right.

(iii) Find ε. [2]

(iv) Show that the spot is at P, the point of its maximum displacement to the right of O, for the fourth time when $t = 12.5$ [4]

When $t = 0$ seconds a second spot on the screen is observed moving horizontally with S.H.M.
It has displacement x_2 metres from O, where

$$x_2 = 4 \sin \frac{9\pi}{25}t$$

The two spots coincide when the first spot is at P for the fourth time and the second spot is at P for the nth time.

(v) Find n. [5]

6 **(a)** A particle P passes an origin O at the start of a straight horizontal track.
A force F newtons acts on P, first accelerating it then decelerating it, where

$$F = 5\pi \sin\left(\frac{\pi x}{12}\right)$$

and x metres is the distance from O that P has moved along the track.

(i) Show that the work done by F over the first 8 m of motion is 90 J. [4]

(ii) Find how far P will be from O when the total work done on it by F has **reduced** to 60 J. [4]

(b) A particle Q of mass m kg is initially at rest at an origin O.
It is acted on by a set of forces whose resultant is a **constant** force **F** newtons.

(i) Clearly indicating the significance of **F.F**, show that W joules, the work done by **F** during the first t seconds of the motion of Q, is

$$W = \frac{\mathbf{F}.\mathbf{F}t^2}{2m}$$ [4]

(ii) If $m = 25$, $\mathbf{F} = (4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$ and $W = 100$, find t. [3]

THIS IS THE END OF THE QUESTION PAPER