Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2013

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

AMF11

MONDAY 24 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(i)** Describe the transformation given by the matrix $\mathbf{M} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ [2]

The matrix $\mathbf{N} = \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}$

The matrix **S** represents the combined effect of the transformation represented by **N** followed by the transformation represented by **M**

(ii) Find the matrix **S** [3]

A region R is mapped to a new region Q under the transformation represented by **S**

(iii) If the area of R is 8 cm^2, find the area of Q. [3]

2 The matrix **P** is given by

$$\mathbf{P} = \begin{pmatrix} 3 & 2 & 2 \\ 0 & 1 & -1 \\ 4 & 0 & 2 \end{pmatrix}$$

(i) Show that the eigenvalues of **P** are –1, 2 and 5 [7]

(ii) For the eigenvalue –1, find a corresponding eigenvector. [4]

3 The permutations

$$e = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \qquad f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$$

$$g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix} \qquad h = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$

form a group G under composition.

(i) Draw up a table for the group G under composition of permutations. [5]

(ii) Write down a subgroup of order 2 [2]

The set $\{1, -1, i, -i\}$, where $i^2 = -1$, forms a group H under multiplication.

(iii) Draw up the group table for H. [3]

(iv) Determine whether the groups G and H are isomorphic. Justify your answer. [2]

4 **(a)** Let $\mathbf{A} = \begin{pmatrix} 1 & 5 \\ -2 & 3 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(i) Verify that $\mathbf{A}^2 = 4\mathbf{A} - 13\mathbf{I}$ [4]

(ii) Hence, or otherwise, express the matrix $\mathbf{A}^3$ in the form $\alpha\mathbf{A} + \beta\mathbf{I}$ where α and β are real numbers. [4]

(b) Let

$$\mathbf{B} = \begin{pmatrix} 1 & -2 & -2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{pmatrix}$$

(i) Find the matrix $\mathbf{B}^2$ [3]

(ii) Hence, or otherwise, find the matrix $\mathbf{B}^{-1}$ [3]

5 **(a)** **(i)** Simplify the complex number

$$\frac{1 + 3\text{i}}{2 + \text{i}}$$

giving your answer in the form $a + b\text{i}$, where a and b are real numbers. [4]

(ii) The complex number $z = x + \text{i}y$ has complex conjugate $z^* = x - \text{i}y$.
If

$$z + 4z^* = \frac{1 + 3\text{i}}{2 + \text{i}}$$

find the exact values of x and y. [4]

(b) On an Argand diagram sketch the locus of those points w which satisfy

$$2 \leqslant | w - (3 + 2\text{i}) | \leqslant 3$$ [5]

6 **(a)** **(i)** Find the centres and radii of the circles

$$x^2 + y^2 + 6x - 8y + 9 = 0$$

$$x^2 + y^2 - 4x + 2y - 4 = 0$$ [4]

(ii) Hence, or otherwise, show that these circles do not intersect. [4]

(b) Two tangents to the circle

$$x^2 + y^2 - 30x - 10y + 225 = 0$$

pass though the origin.

Find the equations of these tangents. [9]

THIS IS THE END OF THE QUESTION PAPER