Rewarding Learning

ADVANCED
General Certificate of Education
2013

Mathematics

Assessment Unit M4

assessing

Module M4: Mechanics 4

[AMM41]

FRIDAY 21 JUNE, MORNING

AMM41

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take g = 9.8 m s^{-2}, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **Fig. 1** below shows a framework ABCDE of seven **equal** light pin jointed rods AB, AC, AE, BC, CE, CD and DE in the shape of a trapezium. BCD is horizontal. The framework rests on two smooth rigid supports at B and D and carries vertical loads of 600 N at A and 200 N at E. The supports give vertical upthrusts of P newtons at B and Q newtons at D.

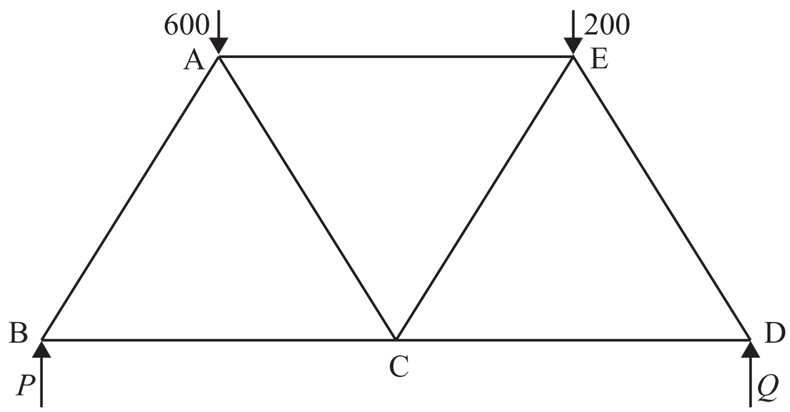

Fig. 1

(i) Show that $P = 500$ [4]

(ii) By resolving in a suitable direction at A and at one other point find the thrust in the rod AC. [4]

2 A uniform lamina of density d kg m^{-2} and mass $36ad$ kg is bounded by the curve $y = ax(6 - x)$ and the x-axis as shown in **Fig. 2** below.

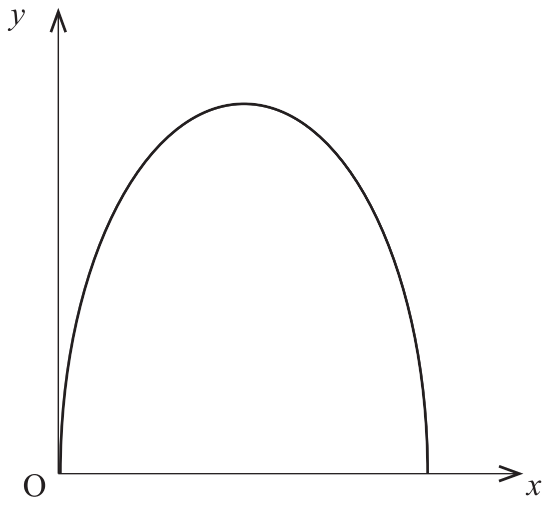

Fig. 2

(i) Show that the y coordinate of the centre of mass, G, is $3.6a$ [7]

G is the point $(b, 18)$.

(ii) Find a and b. [2]

3 Antonin is helping to design a rocket to defend his new world against an attack by the Izans. He believes that the accelerating force F of the rocket depends on the properties of the fuel used. These properties are the Purl P, Swirl S, Twirl T and the Whirl W whose dimensions are shown in **Table 1** below.

Table 1

PROPERTY	DIMENSIONS
P	[MLT]
S	$[\mathrm{ML}^{-2}]$
T	$[\mathrm{M}^{-1}\mathrm{T}^{-1}]$
W	$[\mathrm{ML}^{-1}\mathrm{T}]$

Antonin believes that

$$F = kP^aS^bT^cW^d$$

where k is a dimensionless constant.

(i) Use the Method of Dimensions to show that $b = 3$ and find c and d in terms of a. [7]

When a new fuel, all of whose properties are twice the strength of the originals, is tested F is halved.

(ii) Find the value of a. [4]

When $P = 5$, $S = 4$, $T = 25$ and $W = 2$, $F = 10$

(iii) find the value of k. [2]

4 A particle P of mass 1 kg is moving in a straight horizontal line at 10 $\mathrm{m\,s^{-1}}$ when it impacts directly on a particle Q of mass m kilograms which is at rest. Q moves forward at $3v$ $\mathrm{m\,s^{-1}}$ in the original direction of motion of P and P rebounds at v $\mathrm{m\,s^{-1}}$

(i) Show that $m = \dfrac{10 + v}{3v}$ [3]

As a result of the collision **only 76%** of the initial energy **is conserved**.

(ii) Find v. [6]

The coefficient of restitution between the particles is e.

(iii) Find e. [3]

5 The centre O of the regular hexagon ABCDEF is 0.5m from the mid point of each side and AF is horizontal.
A pair of forces each of magnitude 10 N act along the sides BA and ED.
A force of P newtons acts along AF, one of Q newtons acts along the diagonal BE and one of R newtons acts along CD as shown in **Fig. 3** below.

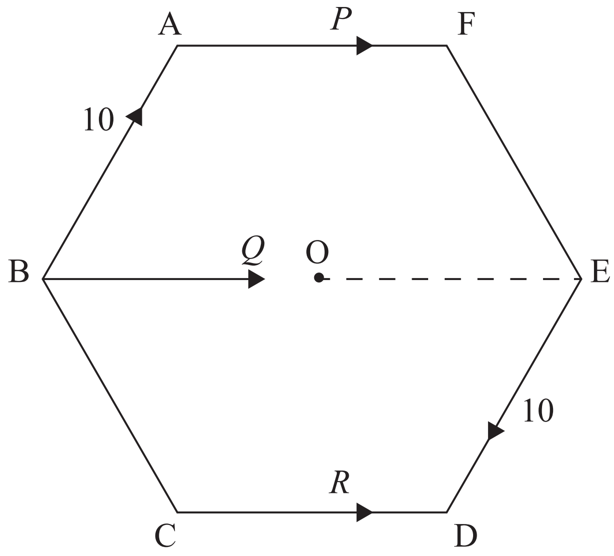

Fig. 3

(i) Show that the vertical component of the resultant of this system of forces is zero. [1]

(ii) Find, in terms of P, Q and R, the horizontal component H of the resultant of this system of forces. [2]

(iii) Find, in terms of P and R, the moment M of this system about O. [2]

(iv) In each of the cases **(a)**, **(b)** and **(c)** in **Table 2** below, find H and M and determine whether the system will reduce to

- a single force
- a couple
- a single force together with a couple.

Table 2

	P	Q	R
(a)	10	0	–10
(b)	–10	20	10
(c)	10	–30	10

[4]

6 **Fig. 4** below represents the forces acting on a car with centre of mass G. The car is negotiating a bend of circular radius r metres, in a road. The road is banked at an angle α to the horizontal. G is h metres above the road and w metres from each side of the car.

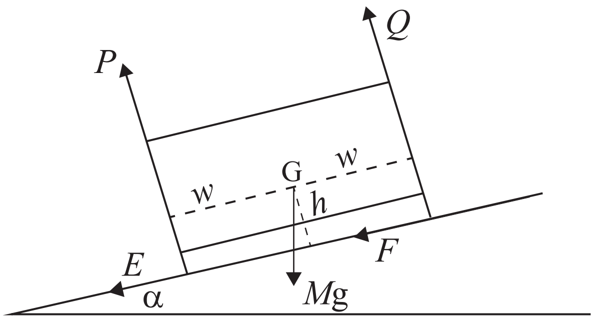

Fig. 4

Zygmunt has been trying to revise his Mechanics notes but has omitted some of the details given by his teacher and has made one transcription error.
In his notes Zygmunt's equations of motion for the above diagram are:

Resolving: $(P + Q)\cos\alpha = Mg + (E + F)\sin\alpha$ (1)

Resolving: $(P + Q)\sin\alpha + (E + F)\cos\alpha = \dfrac{Mv}{r}$ (2)

Moments about G: $Qw = Pw + (E + F)h$ (3)

(i) Identify precisely the forces Q and E. [2]

(ii) Identify the direction for resolving in equation (2). [1]

(iii) Correct the error in equation (2). [1]

(iv) Explain briefly why moments should be taken about G in equation (3). [1]

A car with $h = w$ enters a circular **unbanked** bend of radius r at 20 m s^{-1} and starts to topple outwards.

(v) Use equations (1) and (3) and the corrected (2) to find r. [5]

7 In a distant galaxy Cea is a planet of the star Desta.
Cea has mass m and Desta has mass M.
The distance between the centres of these two celestial bodies is r.
In a simple model Cea moves in a circular orbit around the centre of Desta.
The orbital period of Cea is T_1

(i) Find T_1 in terms of M, r and G the universal gravitational constant. [5]

In a more accurate model Cea and Desta both move in coplanar circular orbits about their mutual centre of mass.

Cea is a distance $\frac{Mr}{(M+m)}$ from this centre of mass.

The orbital period of Cea is now T_2

(ii) Find T_2 in terms of m, M, r and G. [3]

(iii) What distance is Desta from their mutual centre of mass? [1]

(iv) Show that Cea and Desta both have the same orbital period about their mutual centre of mass. [2]

(v) Compare the two models by calculating the ratio $T_1 : T_2$ given that $M = 20m$. [3]

THIS IS THE END OF THE QUESTION PAPER