Rewarding Learning

ADVANCED
General Certificate of Education
2013

Mathematics

Assessment Unit S4

assessing

Module S2: Statistics 2

[AMS41]

FRIDAY 21 JUNE, MORNING

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

				AVAILABLE MARKS
1	(i)	• Assigning employees to values 01–75	M1	
		• How to navigate around the table	M1	
		• Dealing with duplicates	M1	
		• Stop when 6 different values achieved	M1	
		• Dealing with numbers outside the range	M1	
	(ii)	• Listing 2 digit numbers	MW1	
		• Rejecting outside range or duplicates	W1	
		• Final 6 selected	W1	8
2	(i)	$y = a + bx$		
		$b = \dfrac{\sum xy - \dfrac{\sum x \cdot \sum y}{n}}{\sum x^2 - \dfrac{(\sum x)^2}{n}}$	M1	
		$= \dfrac{848320 - \dfrac{270 \times 16528}{6}}{13900 - \dfrac{270^2}{6}}$	W1	
		$= 59.7$ (3 s.f.)	W1	
		$a = \bar{y} - b\bar{x}$	M1	
		$= \dfrac{16528}{6} - (59.7\ldots) \times \dfrac{270}{6}$		
		$= 66.0$ (3 s.f.)	W1	
		So $y = 66.0 + 59.7x$	W1	
	(ii)	$x = 45$ $\quad \hat{y} = 66.0 + 59.7 \times 45$		
		$\hat{y} = 2754.667\ldots [2752.5]$	M1	
		$\hat{y} = 2750$ (3 s.f.)	W1	8

			AVAILABLE MARKS
3 **(i)**	$n = 15 - 1 + 2 = 16$		
	$\sum x = 458 - 24 + 28 + 31 = 493$	MW1	
	$\sum y = 287 - 22 + 17 + 18 = 300$	MW1	
	$\sum x^2 = 14148 - 24^2 + 28^2 + 31^2 = 15317$	MW1	
	$\sum y^2 = 5595 - 22^2 + 17^2 + 18^2 = 5724$	MW1	
	$\sum xy = 8818 - (24 \times 22) + (28 \times 17) + (31 \times 18) = 9324$	MW1	
(ii)	$r = \dfrac{\sum xy - \frac{\sum x . \sum y}{n}}{\sqrt{\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)\left(\sum y^2 - \frac{(\sum y)^2}{n}\right)}}$	M1	
	$r = \dfrac{9324 - \frac{493 \times 300}{16}}{\sqrt{\left(15317 - \frac{493^2}{16}\right)\left(5724 - \frac{300^2}{16}\right)}}$	MW1 M1	
	$r = 0.717$ (3 s.f.)	W1	
(iii)	Moderate positive correlation	M1	10
4 **(i)**	$\hat{\sigma}^2 = \frac{1}{41}\left(3429.15 - \frac{372.9^2}{42}\right)$	M1	
	$= 2.886115\ldots$	W1	
	C.I. $= \bar{x} \pm 1.96\sqrt{\frac{\hat{\sigma}^2}{n}}$	M1	
	$= \frac{372.9}{42} \pm 1.96\sqrt{\frac{2.886\ldots}{42}}$	W1 W1	
	C.I. = (8.36, 9.39) (3 s.f.)	W2	
(ii)	Mean might be different because of a different set of values. The interval would be narrower as n is greater.	M2 M2	11

AVAILABLE MARKS

5 **(i)** In a hypothesis test where a parameter's value is examined a one-tailed test is used to test if the value has differed in one direction only (either upwards or downward). M2

(ii) $\bar{x} = \dfrac{408.1}{40} = 10.2025$

$\hat{\sigma} = \sqrt{\dfrac{1}{39}\left(4183.89 - \dfrac{408.1^2}{40}\right)}$ M1

$= 0.721 \quad (3\text{ s.f.})$ W1

$H_0: \ \mu = 10$ M1

$H_1: \ \mu > 10$ M1

One-tailed test M1

$Z_{\text{crit}} = 1.645$ MW1

$Z_{\text{test}} = \dfrac{\bar{x} - \mu}{\hat{\sigma}/\sqrt{n}}$ M1

$= \dfrac{10.2025 - 10}{0.721/\sqrt{40}}$ M1

$= 1.78 \ (3\text{ s.f.})$ W1

Since 1.78 > 1.645 we reject H_0 and conclude that there is sufficient evidence to suggest that the fat content of the minced beef is greater than 10%. We reject the producer's claim. M2 13

			AVAILABLE MARKS
6	**(i)** $X \sim N(11, 1.5)$ $\quad Y \sim N(8, 1)$		
	Let $S = X + Y \rightarrow S \sim N(11 + 8, 1.5 + 1)$ $S \sim N(19, 2.5)$	MW1	
	$P(X + Y > 21) = P(S > 21) = P\left(Z > \frac{21 - 19}{\sqrt{2.5}}\right)$		
	$= P(Z > 1.265)$	MW1	
	$= 1 - \Phi(1.265)$	M1	
	$= 1 - 0.8971$	W1	
	$= 0.103$ (3 s.f.)	W1	
	(ii) Let $T = 2X - 3Y$	M1	
	$T \sim N(2 \times 11 - 3 \times 8, 2^2 \times 1.5 + 3^2 \times 1)$		
	$T \sim N(-2, 15)$	MW1 M2 W1	
	$P(2X < 3Y) = P(T < 0) = P\left(Z < \frac{0 - (-2)}{\sqrt{15}}\right)$		
	$= P(Z < 0.516)$	MW1	
	$= \Phi(0.516)$		
	$= 0.697$ (3 s.f.)	MW1	12

			AVAILABLE MARKS
7	Let d = reading after – reading before	M1	
	$\sum d = -2.1$		
	$\sum d^2 = 1.39$	MW1	
	$\overline{d} = -0.21$		
	$S_d = \sqrt{\frac{1}{9}\left(1.39 - \frac{(-2.1)^2}{10}\right)} = 0.3247$	MW1	
	$H_0: \mu_d = 0$	M1	
	$H_1: \mu_d < 0$	M1	
	1-tailed t-test with 9 degrees of freedom	M2	
	$t_{crit} = -1.833$	MW1	
	$t_{test} \frac{\overline{d} - 0}{S_d/\sqrt{n}}$	M1	
	$= \frac{-0.21 - 0}{0.3247\ldots/\sqrt{10}} = -2.05$	MW1 M1	
	Since $t_{test} < t_{crit}$ we reject H_0 and conclude that there is sufficient evidence to suggest that the drug lowers cholesterol levels.	M1 M1	13
		Total	**75**