Rewarding Learning

ADVANCED
General Certificate of Education
2013

Mathematics

Assessment Unit M4

assessing

Module M4: Mechanics 4

[AMM41]

FRIDAY 21 JUNE, MORNING

MARK SCHEME

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

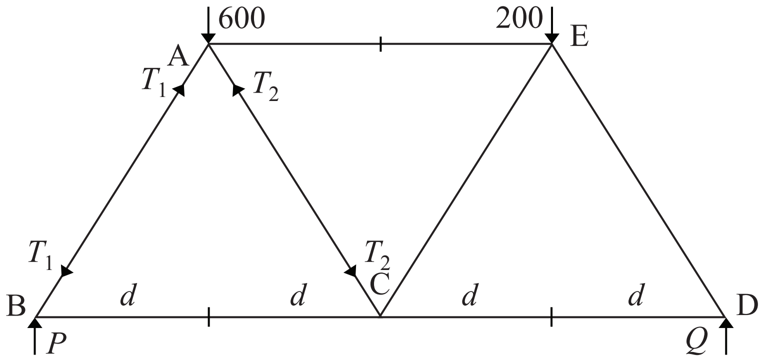
(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

			AVAILABLE MARKS

1 **(i)**



$M(\text{D})$ — M1

$4Pd = 200d + 1800d$ — MW2

$= 2000d$

$P = 500$ N — W1

(ii) ↕@ B $\quad T_1 \cos 30° = P = 500$ — M1 W1

↕@ A $\quad T_1 \cos 30° + T_2 \cos 30° = 600$ — MW1

$$T_2 \frac{\sqrt{3}}{2} = 100$$

$$T_2 = \frac{200\sqrt{3}}{3} \text{ N}$$ — W1 — **8**

2 **(i)** A is (6, 0) — MW1

$$M = \frac{1}{2} d \int_0^6 y^2 \, dx$$ — M1 M1

$$= \frac{1}{2} a^2 d \int_0^6 (6x - x^2)^2 dx$$ — MW1

$$= \frac{1}{2} a^2 d \int_0^6 (36x^2 - 12x^3 + x^4) dx$$

$$= \frac{1}{2} a^2 d \left[12x^3 - 3x^4 + \frac{x^5}{5}\right]_0^6$$ — MW1

$$= \frac{216 a^2 d}{2} \left(12 - 18 + 7\frac{1}{5}\right)$$

$$= 108 a^2 d \times \frac{6}{5}$$ — W1

$$\therefore \bar{y} = \frac{M}{m}$$

$$= \frac{18a}{5} = 3.6a$$ — MW1

(ii) $(b, 18) = \quad (3, 3.6a)$ by symmetry — MW1

$\therefore a = 5, b = 3$ — MW1 — **9**

		AVAILABLE MARKS
3 **(i)** $[MLT^{-2}] = [MLT]^a[ML^{-2}]^b[M^{-1}T^{-1}]^c[ML^{-1}T]^d$	M1	
$= [M]^{a+b-c+d}\,[L]^{a-2b-d}\,[T]^{a-c+d}$		
$\therefore a + b - c + d = 1$ ①	MW1	
$a - 2b \quad - d = 1$ ②	MW1	
$a \quad - c + d = -2$ ③	MW1	
① − ③ $b = 3$	W1	
$\therefore$ ② $\Rightarrow$ $d = a - 7$	W1	
$\therefore$ ③ $\Rightarrow$ $c = 2a - 5$	W1	
(ii) $\frac{1}{2}F = 2^a \times 2^3 \times 2^{2a-5} \times 2^{a-7} \times F$	M1 W1	
$2^{-1} = 2^{4a-9}$	MW1	
$\therefore a = 2$	W1	
(iii) $10 = k \times 25 \times 64 \times \frac{1}{25} \times \frac{1}{32}$	M1	
$= 2k$		
$k = 5$	W1	13
4 **(i)** $10 \rightarrow$ ① $\rightarrow -v$; $\rightarrow 0$ (m) $\rightarrow 3v$; $\rightarrow +$		
Conservation of momentum	M1	
$3mv - v = 10$	W1	
$m = \frac{10 + v}{3v}$	W1	
(ii) $\frac{1}{2} \times m \times 9v^2 + \frac{1}{2} \times 1 \times v^2 = \frac{76}{100} \times \frac{1}{2} \times 1 \times 100$	M1 MW2	
$(9m + 1)v^2 = 76$		
$\left(\frac{30 + 4v}{v}\right)v^2 = 76$	MW1	
$4v^2 + 30v - 76 = 0$		
$2v^2 + 15v - 38 = 0$	MW1	
$(v - 2)(2v + 19) = 0$		
$v > 0 \quad \therefore \quad v = 2$	W1	
(iii) By Newton's Law	M1	
$3v - (-v) = -e(0 - 10)$	W1	
$4v = 10e$		
$10e = 8$		
$e = 0.8$	W1	12

				AVAILABLE MARKS
5	**(i)** Resolving vertically $10 \sin 60° - 10 \sin 60° = 0$		MW1	
	(ii) Resolving horizontally $P + Q + R + 10 \cos 60° - 10 \cos 60° = P + Q + R$		M1 W1	
	(iii) Moments about O $(P + 10 - R + 10) \times 0.5 = 0.5(P-R)+10$		M1 W1	

(iv)

	Horizontal force $P + Q + R$	**Couple 0.5 $(P - R) + 10$**	**System**		
(a)	0	20	Couple 20 Nm clockwise	MW1	
(b)	20	0	Single force 20 N (dir AF)	MW1	
(c)	–10	10	Couple 10 Nm clockwise and force 10 N (dir FA)	MW2	9

6 **(i)** Q is the reaction of the road on the outer wheels — MW1
E is the friction between the road and the inner wheels — MW1

(ii) Resolving horizontally — MW1

(iii) R.H.S $= \dfrac{Mv^2}{r}$ — MW1

(iv) The moments of the forces balance at this point — MW1

(v) $\alpha = 0, \alpha = 0, \quad P = E = 0$ — M1 M1

$\therefore$ ① → $Q = Mg$

② → $F = \dfrac{Mv^2}{r}$

③ → $Q = F$ — MW1

$\therefore$ → $\dfrac{Mv^2}{r} = Mg$ — M1

so $r = \dfrac{v^2}{g} = \dfrac{400}{9.8} = 40.8$ m (3 s.f.) — W1 — 10

			AVAILABLE MARKS
7	**(i)** $T = \dfrac{2\pi}{\omega}$	M1	
	$mr\omega^2 = \dfrac{GMm}{r^2}$	M1 M1	
	$\omega = \sqrt{\dfrac{GM}{r^3}}$	W1	
	$T_1 = 2\pi\sqrt{\dfrac{r^3}{GM}}$	W1	
	(ii) $m\left(\dfrac{M}{M+m}\right)r\omega_2^2 = \dfrac{GMm}{r^2}$	M1 W1	
	$\omega_2^2 = \dfrac{G(M+m)}{r^3}$		
	$T_2 = 2\pi\sqrt{\dfrac{r^3}{G(M+m)}}$	MW1	
	(iii) $Md = m\left(\dfrac{Mr}{M+m}\right)$		
	$d = \dfrac{mr}{(M+m)} = \left(\dfrac{m}{M+m}\right)r$	MW1	
	(iv) $M\left(\dfrac{m}{M+m}\right)r\omega_3^2 = \dfrac{GMm}{r^2}$	MW1	
	$\omega_3^2 = \dfrac{G(M+m)}{r^3}$		
	$\therefore \quad T_3 = T_2$	MW1	
	Alternative solution		
	Cea, Desta and the centre of mass are coplanar	MW1	
	$\therefore$ Desta has the same period as Cea	MW1	
	(v) $M = 20m$ $\quad T_1 = 2\pi\sqrt{\dfrac{r^3}{20mG}}$		
	$T_2 = 2\pi\sqrt{\dfrac{r^3}{21mG}}$	MW1	
	$T_2 : T_1 = \dfrac{1}{\sqrt{21}} : \dfrac{1}{\sqrt{20}}$		
	$= \sqrt{20} : \sqrt{21}$	W1	
	Here the simple model is a good approximation to the more accurate one.	MW1	14
		Total	**75**