Rewarding Learning

ADVANCED
General Certificate of Education
January 2014

Mathematics

Assessment Unit C3

assessing

Module C3: Core Mathematics 3

[AMC31]

MONDAY 20 JANUARY, MORNING

AMC31

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Solve $|3x + 7| \geqslant 2$ [4]

2 **(i)** By completing the square, write $x^2 - 4x + 12$ in the form

$$(x + a)^2 + b$$ [2]

(ii) Hence describe two successive transformations which map the graph of the function $y = x^2$ to the graph of the function $y = x^2 - 4x + 12$ [2]

3 Expand

$$\frac{1}{(4 - x)^2}$$

in a binomial series up to and including the term in x^3 [7]

4 Express

$$\frac{4x^2 - x + 7}{(2x - 1)(x + 2)}$$

in partial fractions. [8]

5 **(a)** A curve is described by the parametric equations

$$x = 2\cot\theta \qquad y = 3 - \sin\theta$$

(i) Find a Cartesian equation connecting x and y. [5]

(ii) Find the points where the curve crosses the y-axis. [2]

(b) **(i)** Differentiate

$$\frac{\ln x - 1}{3x^2}$$

simplifying your answer. [4]

(ii) Find

$$\int \frac{(e^{2x} - e^{-x})^2}{e^x}\,dx$$ [6]

6 **(a)** Evaluate

$$\sec^2 x + \sin^2 x + \operatorname{cosec}^2 x + \cos^2 x - \tan^2 x - \cot^2 x$$ [2]

(b) Solve

$$7\sec\theta - 8 = \frac{2}{\cot^2\theta} \qquad -\pi \leqslant \theta \leqslant \pi$$ [8]

7 A flower bed is modelled as the area bounded by the curve $y = 1 + \sin x$, the axes and the line $x = p$ as shown shaded in **Fig. 1** below:

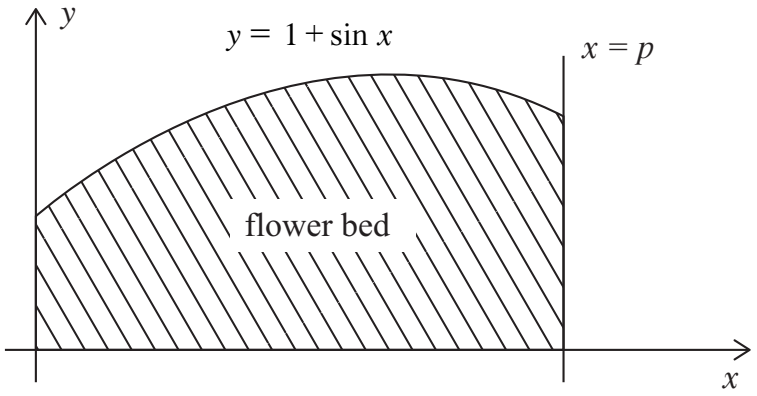

Fig. 1

(i) Show that the area of this flower bed is

$$p - \cos p + 1$$ [6]

(ii) The flower bed is to have an area of 4 units^2
Using the Newton–Raphson method, taking the starting value of $p_0 = 2$, find an improved value for p after two applications of the method. [6]

8 Consider the function $y = \text{cosec}^2 x$.

(i) Find $\dfrac{d^2y}{dx^2}$, giving your answer in terms of cosec x. [8]

(ii) Hence find the turning points of the curve

$$y = \text{cosec}^2 x \quad 0 \leqslant x \leqslant 2\pi$$

and determine their nature. [5]

THIS IS THE END OF THE QUESTION PAPER