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**ADVANCED**
**General Certificate of Education**
**2014**

# Mathematics

## Assessment Unit F2

*assessing*

## Module FP2: Further Pure Mathematics 2

**[AMF21]**

AMF21

**THURSDAY 12 JUNE, AFTERNOON**

**TIME**

1 hour 30 minutes.

**INSTRUCTIONS TO CANDIDATES**

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

**INFORMATION FOR CANDIDATES**

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$



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**Answer all eight questions.**

**Show clearly the full development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

**1** Given that

$$f(x) = \ln (1 + \sin x)$$

find the Maclaurin expansion for $f(x)$ up to and including the term in $x^2$ [6]

**2** Find, in radians, the general solution of the equation

$$\sin 3\theta = \sin \theta$$ [6]

**3** A geometric series is given by

$$e^{3x} + 3e^{x} + 9e^{-x} + \ldots \qquad x > 0$$

**(i)** Find an expression for the $n$th term of the series. [3]

**(ii)** Find the range of values of $x$ for which the series has a sum to infinity. [4]

**(iii)** Find the sum to infinity when $x = \ln 2$ [4]

**4** **(i)** Given that

$$xy + 3y - x = 0$$

show that

$$(x + 3)\frac{\mathrm{d}y}{\mathrm{d}x} + y = 1 \qquad [2]$$

**(ii)** Hence, using mathematical induction, prove that for $n \geqslant 2$

$$(x + 3)\frac{\mathrm{d}^n y}{\mathrm{d}x^n} + n\frac{\mathrm{d}^{n-1} y}{\mathrm{d}x^{n-1}} = 0 \qquad [5]$$

**5** **(i)** Express in partial fractions

$$\mathrm{f}(x) = \frac{2}{(x-1)^2(x^2+1)} \qquad [6]$$

**(ii)** Hence find the exact value of

$$\int_2^3 \frac{2}{(x-1)^2(x^2+1)}\,\mathrm{d}x \qquad [5]$$

**6** **(i)** Show that

$$(Z^n - \mathrm{e}^{\mathrm{i}\theta})(Z^n - \mathrm{e}^{-\mathrm{i}\theta}) \equiv Z^{2n} - 2Z^n\cos\theta + 1$$

where $Z$ is a complex number. [2]

**(ii)** Hence or otherwise find in $(\cos\theta + \mathrm{i}\sin\theta)$ form the roots of the equation

$$Z^4 - Z^2\sqrt{2} + 1 = 0 \qquad [5]$$

**7** **(i)** Show that an equation of the normal to the parabola $y^2 = 4ax$ at the point $P(at^2, 2at)$ is

$$y + tx = 2at + at^3$$ [4]

The normal at P meets the curve again at the point $Q(as^2, 2as)$.

**(ii)** Show that if $t \neq 0$

$$s = -\left(t + \frac{2}{t}\right)$$ [3]

**(iii)** Using **(ii)** show that

$$PQ^2 = \frac{16a^2(t^2+1)^3}{t^4}$$ [4]

**8** A particle P moves in a straight line so that its displacement $x$ centimetres from a fixed point at time $t$ seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = \lambda \sin 2t$$

where $\lambda$ is a constant.

**(i)** Given that $x = 0$ and $\frac{dx}{dt} = \frac{\lambda}{4}$ when $t = 0$, find $x$ in terms of $t$ and $\lambda$. [14]

**(ii)** As $t$ becomes large describe what happens to the solution found in part **(i)**. [2]

**THIS IS THE END OF THE QUESTION PAPER**