Mathematics

Assessment Unit C2

assessing

Module C2: AS Core Mathematics 2

[AMC21]

THURSDAY 28 MAY, MORNING

AMC21

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Three sequences are given below:

A $\quad u_n = n^2 - 1$

B $\quad u_n = \dfrac{3n^2 + 1}{n^2}$

C $\quad u_{n+1} = 4 - u_n \quad u_1 = 6$

For each sequence describe the behaviour of the sequence as $n \to \infty$
If a sequence converges, state its limit. [4]

2 A jeweller is making a pendant for a necklace.
It can be modelled as two touching circles as shown in **Fig. 1** below.
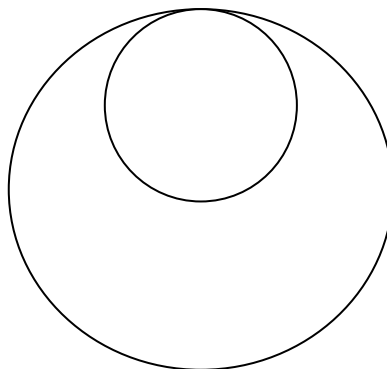

Fig. 1

The equation of the larger circle is

$$x^2 - 6x + y^2 - 8y + 21 = 0$$

(i) Find the centre and radius of this circle. [4]

The diameter of the smaller circle is half the diameter of the larger circle.
The centre of the smaller circle is vertically above the centre of the larger circle.

(ii) Find the equation of the smaller circle. [3]

3 **(a)** A rescue helicopter takes off from its base and flies for 70 km on a bearing of 110° to rescue an injured sailor.
After picking up the sailor, the helicopter flies 80 km, on a bearing of 030° to the nearest hospital.
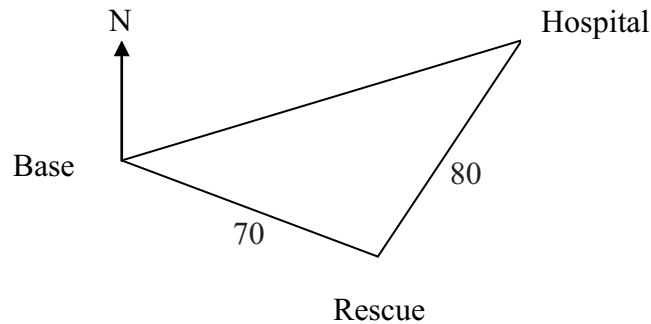The helicopter then returns directly to its base, as shown in **Fig. 2** below.

Fig. 2

(i) Calculate the distance the helicopter flies from the hospital directly back to its base. [3]

(ii) Calculate the bearing of the base from the hospital. [3]

(b) **Fig. 3** below shows part of an ornamental garden in the shape of a sector of a circle, with radius 6 m and angle 1.2 radians.
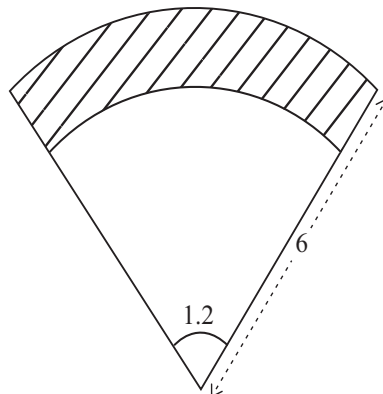

Fig. 3

The shaded area is to be a flowerbed and the unshaded area is to be lawn.
The lawn is also a sector.
The area of the lawn is $\frac{2}{3}$ the area of the total sector.

(i) Find the radius of the sector of the circle which is to be lawn. [4]

(ii) Find the perimeter of the flowerbed. [4]

4 The second term of a Geometric Progression is 270
The fifth term of the Geometric Progression is 80

    (i) Find the first term and the common ratio. [5]

    (ii) Why does a sum to infinity exist for this Geometric Progression? [1]

    (iii) Find the sum to infinity of this Geometric Progression. [2]

5 Find the first three terms, in descending powers of x, in the binomial expansion of

$$\left(2x - \frac{5}{x^2}\right)^9$$ [5]

6 **(a)** **(i)** Sketch the graph of

$$y = \sin 2x$$

        for $0 \leqslant x \leqslant 2\pi$ [2]

    (ii) Use the trapezium rule with 5 ordinates to find an approximate value for

$$\int_0^1 \sin 2x \, dx$$ [5]

    (b) Solve

$$2 \cos \theta = 3 \tan \theta$$

    for $0° \leqslant \theta \leqslant 360°$ [7]

7 **(a)** Integrate

$$\frac{x}{3} + \frac{4}{x^2} - 6\sqrt{x}$$

with respect to *x*. [4]

(b) Find the area enclosed by the curve $y = x^2 - 4x + 3$, the *x*-axis and the
lines $x = 1$ and $x = 4$ [7]

8 **(a)** Sketch the graph of

$$y = 2^x - 5$$

clearly indicating where it crosses the *x*-axis and the *y*-axis. [3]

(b) If $\log_4 x = k$, express the following in terms of k:

(i) $\log_4 x^3$ [1]

(ii) $\log_4 \frac{\sqrt[3]{x}}{16}$ [2]

(iii) $\log_2 x$ [2]

(iv) Hence solve for *x*

$$\log_4 x^3 + \log_4 \frac{\sqrt[3]{x}}{16} = \log_2 x$$ [4]

THIS IS THE END OF THE QUESTION PAPER