Rewarding Learning

ADVANCED
General Certificate of Education
2015

Mathematics

Assessment Unit M4

assessing

Module M4: Mechanics 4

AMM41

[AMM41]

WEDNESDAY 24 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Three spheres A, B and C of equal radius and masses m, $4m$ and $2m$ respectively are at rest in a straight line on a smooth horizontal surface. A is projected towards B with speed u and collides directly with it. As a result of the collision A is brought to rest.

(i) Show that the coefficient of restitution between A and B is $\frac{1}{4}$ [5]

A collision now occurs between B and C.

The coefficient of restitution between B and C is $\frac{4}{5}$

(ii) Find, in terms of u, the velocities of B and C after this collision. [4]

(iii) Find, in terms of u and m, the total loss of kinetic energy due to the two collisions. [3]

2 **Fig. 1** below shows a framework of five light pin-jointed rods AB, BC, CD, DA and DB.
The framework supports a weight of 20 N at A.
The rods all lie in the same vertical plane.
BC = CD = DA = DB = 50 cm.
The rod AD is horizontal.
The framework is freely hinged to a vertical wall at B and the pin at C rests against the wall, making a smooth contact.

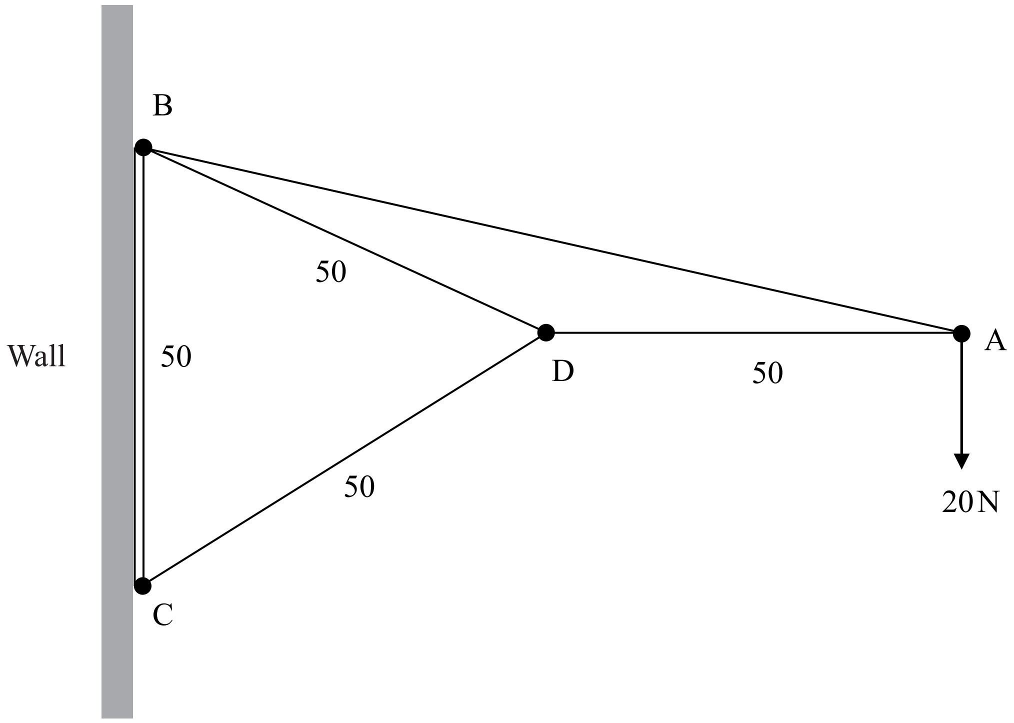

Fig. 1

(i) Explain briefly why the reaction on the pin at C must be horizontal. [1]

(ii) Show that the magnitude of the reaction at C is approximately 37.3 N. [3]

(iii) Find the forces in the rods CD, CB, DA and AB. [5]

(iv) State which of these rods could be replaced by light ropes, giving a reason for your answer. [2]

3 **Fig. 2** below shows the relative positions of the Moon, M, Earth, E, and Sun, S, represented as particles, lying in a straight line with the Earth between the Sun and the Moon during a lunar eclipse.
The distance between S and E is R.
The distance between E and M is r.

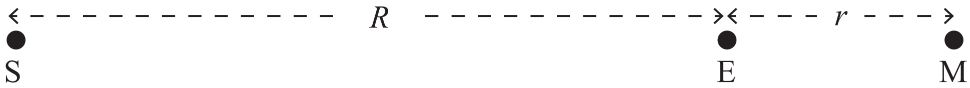

Fig. 2

The masses of the Sun, Earth and Moon are M_S, M_E and M_M respectively.

(i) Write down the expression, in terms of M_S, M_M, R, r and G, the universal gravitational constant, for the gravitational force on the Moon due to the Sun. [1]

The magnitude of the gravitational force between E and M is F_{EM}
The magnitude of the gravitational force between S and M is F_{SM}

(ii) Show that $\frac{F_{EM}}{F_{SM}} = \frac{M_E}{M_S}\left(\frac{R}{r} + 1\right)^2$ [2]

Given that $M_S = 3.328 \times 10^5 M_E$ and $R = 3.896 \times 10^2 r$,

(iii) find the value of $\frac{F_{EM}}{F_{SM}}$ [2]

During a solar eclipse M, E and S lie in a straight line with M between S and E.

(iv) Find the value of $\frac{F_{EM}}{F_{SM}}$ in this case. [3]

4 Take **i** and **j** to be unit vectors in the directions of the axes O*x* and O*y* respectively.

Fig. 3 below shows three forces $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ newtons which act at the points whose coordinates are (2, 5), (3, –4) and (–2, 2) respectively.

$$\mathbf{F_1} = (-2a\,\mathbf{i} + 3\mathbf{j})$$
$$\mathbf{F_2} = (-2\mathbf{i} - 4a\mathbf{j})$$
$$\mathbf{F_3} = -6\mathbf{i}$$

where a is a constant and the unit of distance is metres.

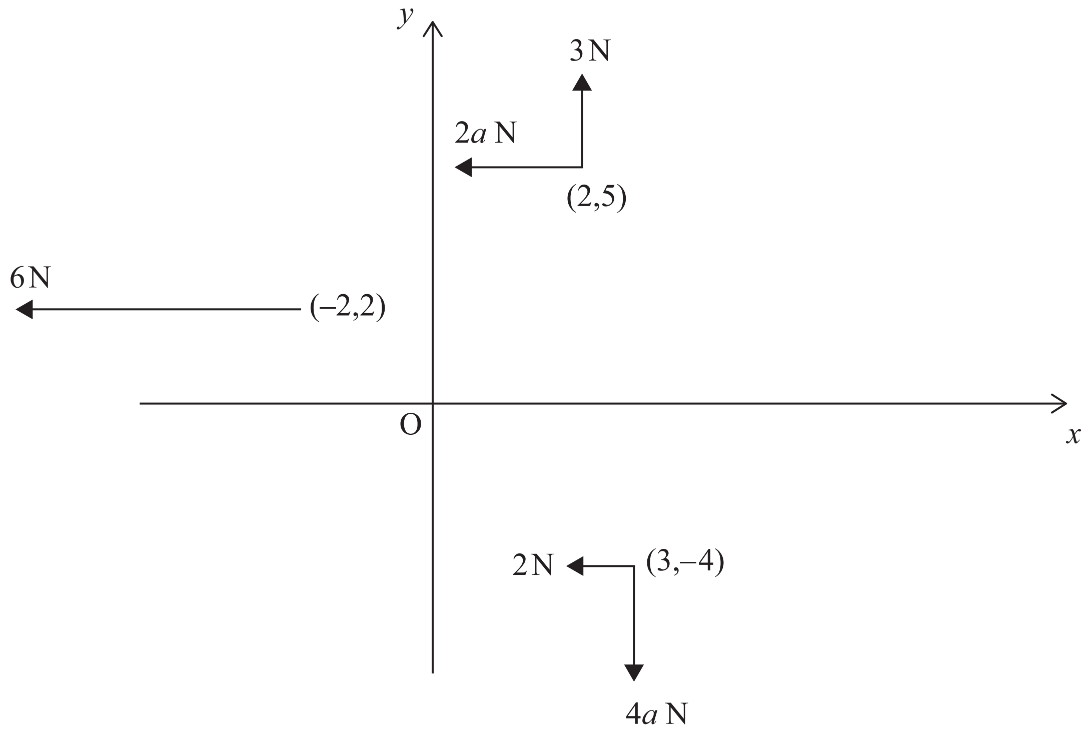

Fig. 3

(i) Show that the sum of the moments of $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ about the origin can be expressed as $(10 - 2a)$ N m anticlockwise. [4]

(ii) If the system of forces reduces to a single force acting at the origin, write down the value of a. [1]

(iii) If, instead, the system of forces is equivalent to a force acting at the origin together with a couple of magnitude 16 N m, find the possible values of a. [3]

5 **(i)** Show that the centre of mass, G, of a uniform solid right circular cone of height h and base radius r lies on its axis of symmetry $\frac{h}{4}$ from its base. [6]

Fig. 4 below shows a uniform solid right circular cone of height 15 cm and base radius 8 cm resting on a horizontal surface which is sufficiently rough to prevent slipping. A force P newtons, which just causes the cone to topple, is applied to the vertex of the cone at 20° below the horizontal.

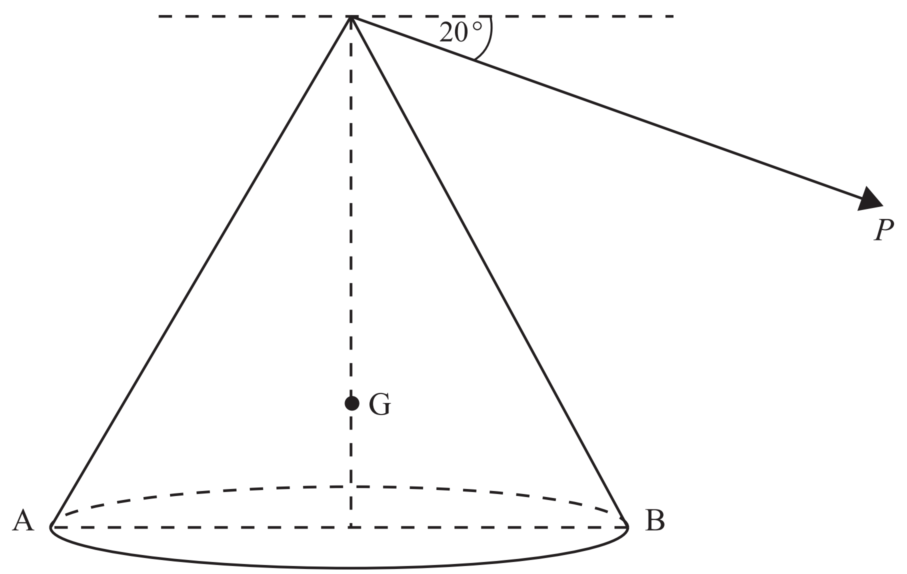

Fig. 4

A diameter of the base, AB, the centre of mass, G, and the force P all lie in the same vertical plane.

The mass of the cone is $\frac{1}{2}$ kg.

(ii) Find P. [4]

6 The velocity v of surface water waves depends on the wavelength λ of the waves, the acceleration due to gravity g, the density of water ρ and the surface tension S of water. The velocity is related to the other variables as follows:

$$v = k\,\lambda^a g^b \rho^c S^d$$

where k is a dimensionless constant.

The dimensions of S are $[MT^{-2}]$

Using the method of dimensions,

(i) show that $c + d = 0$, [5]

(ii) find a and b in terms of d, [3]

(iii) show that a possible relationship between the variables is

$$v = k\sqrt{\lambda g}\left(\frac{S}{\rho\lambda^2 g}\right)^d$$ [2]

For deep water waves, the velocity is independent of S.

(iv) Using the relationship given in **(iii)**, write down a formula for the velocity of deep water waves. [1]

For shallow water waves, the velocity is independent of g.

(v) Again using the relationship given in **(iii)**, find a formula for the velocity of shallow water waves. [2]

7 **Fig. 5** below shows a bead of mass m threaded on a smooth circular wire of radius r fixed in a vertical plane. O is the centre of the circle.
A light inextensible string attached to the bead passes through a smooth ring fixed at O and supports a particle of mass M hanging freely.
The bead is projected with speed u from A, the lowest point on the wire.
When the bead is at P, OP makes an angle θ with the downward vertical and the speed of the bead is v.

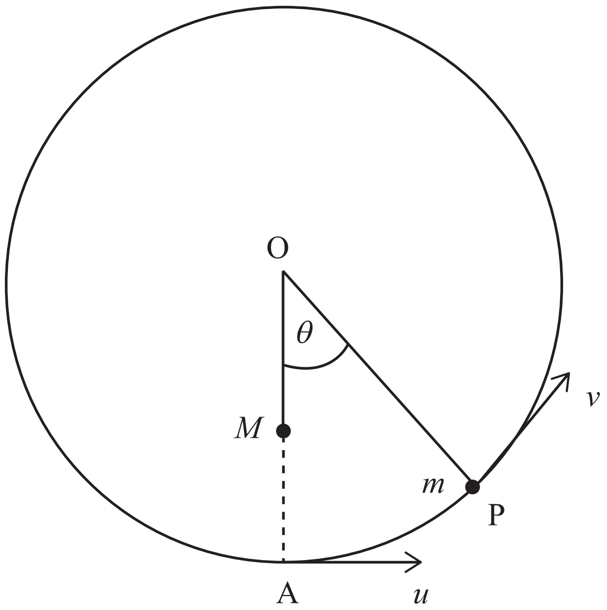

Fig. 5

Take the gravitational potential energy to be zero at A.

Given that $u^2 = 8gr$,

(i) show that the bead makes complete revolutions, [4]

(ii) find, in terms of M, m, g and θ, an expression for the reaction of the wire on the bead at P, [5]

(iii) show that if there is no reaction between the bead and the wire at some points of the motion then $3m \leqslant M \leqslant 9m$. [4]

THIS IS THE END OF THE QUESTION PAPER