Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2015

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

WEDNESDAY 24 JUNE, MORNING

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

		AVAILABLE MARKS
1 **(i)** $\begin{pmatrix} 4 & 9 & 0 \\ 0 & -2 & 8 \\ 0 & 0 & 7 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 12 - 18 + 0 \\ 0 + 4 + 0 \\ 0 + 0 + 0 \end{pmatrix}$	M1	
$= \begin{pmatrix} -6 \\ 4 \\ 0 \end{pmatrix}$	W1	
(ii) Since $\mathbf{R}\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} = -2\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$, then $\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$ is an eigenvector of $\mathbf{R}$		
with a corresponding eigenvalue of –2	MW2	
(iii) $\mathbf{R}^2\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix} = \mathbf{RR}\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$	MW1	
$= -2\mathbf{R}\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$	MW1	
$= -2 \times -2\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$	W1	
$= 4\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$	W1	8

			AVAILABLE MARKS
2	**(i)**	$x^2 + y^2 - 4x - 8y + 10 = 0$ $\Rightarrow (x-2)^2 + (y-4)^2 = 10$	

Hence the circle has centre (2, 4) MW1

$\Rightarrow$ Gradient of radius $= \dfrac{5-4}{-1-2}$

$= -\dfrac{1}{3}$ MW1

Alternative solution:

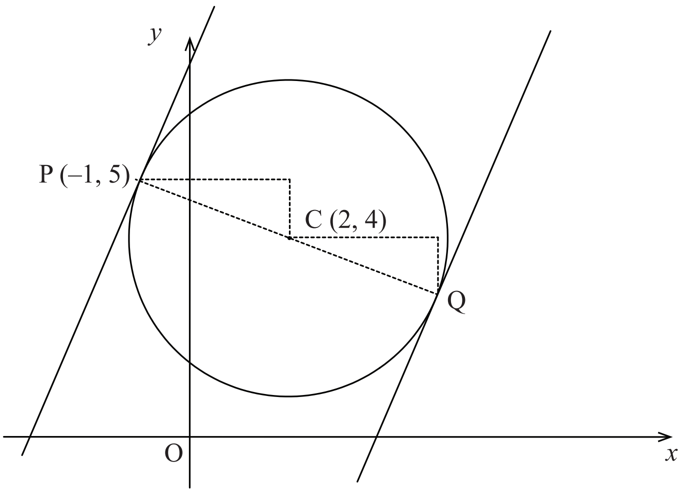
$2x + 2y\dfrac{dy}{dx} - 4 - 8\dfrac{dy}{dx} = 0$ M1 W1

$\therefore \dfrac{dy}{dx} = \dfrac{4-2x}{(2y-8)}$ At pt $(-1, 5)$

$\dfrac{dy}{dx} = \dfrac{4-2(-1)}{10-8} = \dfrac{6}{2} = 3$

Therefore the gradient of the tangent is 3 MW1
$\Rightarrow$ Equation of tangent is $y = 3x + c$
Using the point (–1, 5) gives $5 = -3 + c$ M1
Hence $c = 8$
and the equation of the tangent is $y = 3x + 8$ MW1

(ii)



From the diagram, P to C is 3 right and 1 down MW1
By symmetry, then C to Q is also 3 right and 1 down MW1
Hence Q is the point (5, 3) W1
Since $y = 3x + c$, then using (5, 3) gives
$3 = 15 + c$ M1
$\Rightarrow c = -12$
Hence the other tangent is $y = 3x - 12$ W1

(ii) Alternative Solution

		AVAILABLE MARKS
The parallel tangent is $y = 3x + c$	MW1	
Substitute to give		
$x^2 + (3x + c)^2 - 4x - 8(3x + c) + 10 = 0$	MW1	
$\Rightarrow x^2 + 9x^2 + 6cx + c^2 - 4x - 24x - 8c + 10 = 0$		
$\Rightarrow 10x^2 + x(6c - 28) + (c^2 - 8c + 10) = 0$		
For a tangent, then $B^2 - 4AC = 0$	M1	
Hence, $(6c - 28)^2 = 40(c^2 - 8c + 10)$	W1	
$\Rightarrow 36c^2 - 336c + 784 = 40c^2 - 320c + 400$		
$\Rightarrow 4c^2 + 16c - 384 = 0$		
$\Rightarrow c^2 + 4c - 96 = 0$		
$\Rightarrow (c + 12)(c - 8) = 0$		
$\Rightarrow c = -12, 8$		
Hence the other tangent is $y = 3x - 12$	W1	10

			AVAILABLE MARKS
3	**(i)** $\begin{vmatrix} a & 1 & -2 \\ 1 & 4 & 1 \\ -2 & 1 & a \end{vmatrix} = 0$ for non-unique solution.	M1	
	Hence $a(4a-1)-1(a+2)-2(1+8)=0$	M1 W1	
	$\Rightarrow 4a^2 - a - a - 2 - 18 = 0$		
	$\Rightarrow 4a^2 - 2a - 20 = 0$	W1	
	$\Rightarrow 2a^2 - a - 10 = 0$		
	$\Rightarrow (a+2)(2a-5) = 0$		
	$\Rightarrow a = -2,\ a = 2\frac{1}{2}$	W1	
	(ii) $a = -2$ gives a non-unique solution	MW1	
	Setting up 3 equations	M1	
	$-2x + y - 2z = 6$ ① $x + 4y + z = 2$ ② $-2x + y - 2z = 5$ ③		
	Since ① and ③ are contradictory then there is no solution.	MW1	
	(iii) Setting up 3 equations	MW1	
	$-2x + y - 2z = 5$ ① $x + 4y + z = 2$ ② $-2x + y - 2z = 5$ ③		
	Since ① and ③ are identical, then there are infinitely many solutions.		
	① $-2x + y - 2z = 5$ 2 × ② $2x + 8y + 2z = 4$ Adding gives $9y = 9$	M1	
	Hence $y = 1$	W1	
	$\Rightarrow x + 4 + z = 2$ $\Rightarrow x + z = -2$		
	Therefore, the general solution is $(t, 1, -2 - t)$	MW1	12

4 Closure

AVAILABLE MARKS

$$\begin{pmatrix} r & s \\ s & r \end{pmatrix}\begin{pmatrix} p & q \\ q & p \end{pmatrix} = \begin{pmatrix} rp + sq & rq + sp \\ sp + rq & sq + rp \end{pmatrix}$$ M1 W1

$$= \begin{pmatrix} w & v \\ v & w \end{pmatrix}, \text{ where } w = rp + sq \text{ and}$$

$$v = rq + sp$$

It is also necessary to show that $w^2 \neq v^2$ M1

$w^2 = r^2p^2 + 2rpsq + s^2q^2$ MW1

$v^2 = r^2q^2 + 2rpsq + s^2p^2$

and hence are not equal, since $p^2 \neq q^2$, $r^2 \neq s^2$ W1

Hence the product is of the same format and closure exists. MW1

Identity

Identity for multiplication is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ M1

Test $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} r & s \\ s & r \end{pmatrix} = \begin{pmatrix} r & s \\ s & r \end{pmatrix}$ W1

and $\begin{pmatrix} r & s \\ s & r \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} r & s \\ s & r \end{pmatrix}$

Since $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is of the same format as $\begin{pmatrix} r & s \\ s & r \end{pmatrix}$, then an identity exists. MW1

Alternative Solution

$$\begin{pmatrix} a & b \\ b & a \end{pmatrix}\begin{pmatrix} r & s \\ s & r \end{pmatrix} = \begin{pmatrix} r & s \\ s & r \end{pmatrix}$$ M1

Hence $ra + sb = r$ W1

and $sa + rb = s$

$\Rightarrow r^2a + rsb = r^2$

and $s^2a + rsb = s^2$

Subtracting gives

$(r^2 - s^2)\,a = r^2 - s^2$

$\Rightarrow (r^2 - s^2)(a - 1) = 0$

Hence $a = 1$ since $r^2 \neq s^2$ MW1

Therefore, $b = 0$

Hence identity is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Inverse

AVAILABLE MARKS

The inverse $\begin{pmatrix} r & s \\ s & r \end{pmatrix}$ under matrix multiplication is M1

$$\frac{1}{r^2 - s^2}\begin{pmatrix} r & -s \\ -s & r \end{pmatrix}$$

Since $r^2 \neq s^2$, then the inverse exists and the matrix is of the same W1

format as $\begin{pmatrix} r & s \\ s & r \end{pmatrix}$

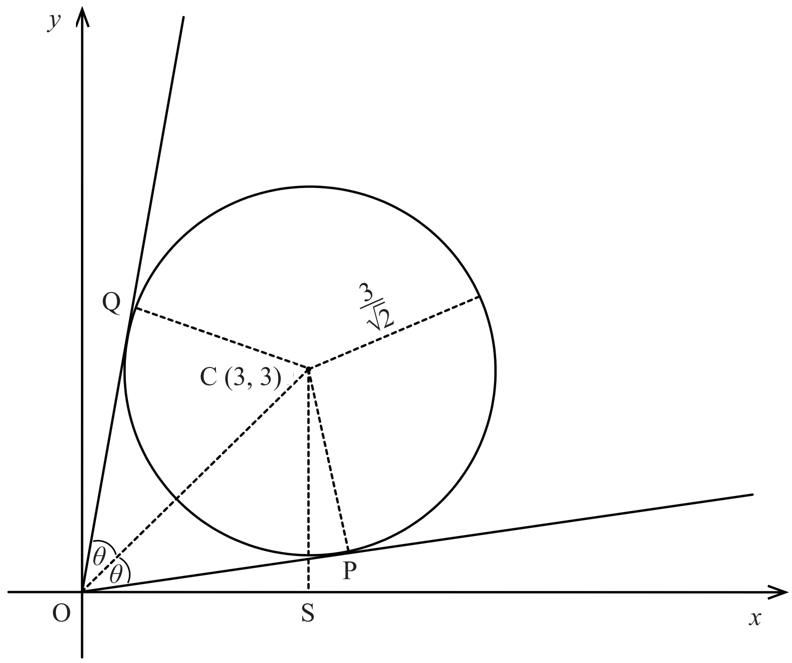
Since the associative law is also true, then S forms a group under matrix multiplication. MW1 12

		AVAILABLE MARKS
5 **(a)** The required matrix is $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$	M1	
$\theta = 45^\circ \Rightarrow$ matrix is $\begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix}$	MW1	
(b) Under the mapping, the line $y = mx$ is mapped to $y = -mx$	MW1	
$\Rightarrow \begin{pmatrix}2 & 1\\ -6 & 3\end{pmatrix}\begin{pmatrix}t\\ mt\end{pmatrix} = \begin{pmatrix}x\\ -mx\end{pmatrix}$	M1 MW1	
$\Rightarrow 2t + mt = x$ $-6t + 3mt = -mx$	W1	
Dividing gives $\frac{2+m}{-6+3m} = \frac{1}{-m}$	M1	
Expanding gives $-2m - m^2 = -6 + 3m$	W1	
$\Rightarrow m^2 + 5m - 6 = 0$		
$\Rightarrow (m+6)(m-1) = 0$		
$\Rightarrow m = -6, m = 1$	W2	10

			AVAILABLE MARKS
6 **(a)** **(i)**	$(a + bi)^2 = -5 + 12i$		
	$\Rightarrow a^2 + 2abi - b^2 = -5 + 12i$	M1	
	Equating real terms: $a^2 - b^2 = -5$ ①	M1 W1	
	Equating imaginary terms: $2ab = 12$ ②	W1	
	Re-write ② to give $b = \frac{6}{a}$	W1	
	Substitute into ① to give	M1	
	$a^2 - \left(\frac{6}{a}\right)^2 = -5$		
	$\Rightarrow a^4 + 5a^2 - 36 = 0$		
	$\Rightarrow (a^2 + 9)(a^2 - 4) = 0$		
	$\Rightarrow a = \pm 2$	W1	
	$\Rightarrow b = \pm 3$	W1	
(ii)	Use quadratic formula to obtain	M1	
	$z = \frac{(4 - i) \pm \sqrt{(4 - i)^2 - 4(5 - 5i)}}{2}$	W1	
	$\Rightarrow z = \frac{(4 - i) \pm \sqrt{16 - 8i - 1 - 20 + 20i}}{2}$		
	$\Rightarrow z = \frac{(4 - i) \pm \sqrt{-5 + 12i}}{2}$	W1	
	$\Rightarrow z = \frac{(4 - i) \pm (2 + 3i)}{2}$	M1	
	$\Rightarrow z = \frac{6 + 2i}{2}, \frac{2 - 4i}{2}$	W1	
	Hence $z = 3 + i, 1 - 2i$	W1	

(b) **(i)** Circle, centre (3, 3) and radius $\frac{3}{\sqrt{2}}$ MW3

AVAILABLE MARKS



(ii) The minimum/maximum values of arg w occur at the points P/Q, where OP/OQ are tangents to the circle. M1

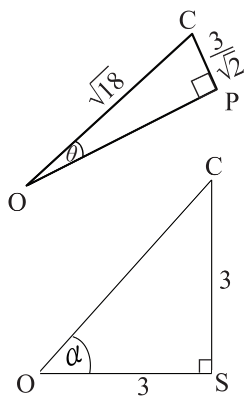
$$OC = \sqrt{3^2 + 3^2}$$
$$= \sqrt{18}$$ MW1

$$\Rightarrow \sin\theta = \frac{\frac{3}{\sqrt{2}}}{\sqrt{18}} = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$ MW1

$$\tan\alpha = \frac{3}{3} = 1$$ MW1

$$\Rightarrow \alpha = \frac{\pi}{4}$$



Minimum value of arg w is $\frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$ MW1

Maximum value of arg w is $2\left(\frac{\pi}{6}\right) + \frac{\pi}{12} = \frac{5\pi}{12}$ MW1

Hence $\frac{\pi}{12} \leqslant \arg w \leqslant \frac{5\pi}{12}$ 23

Total **75**