Rewarding Learning

ADVANCED

General Certificate of Education

2016

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

AMC41

[AMC41]

WEDNESDAY 8 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find

$$\int \frac{x + 4}{x(2 - x)}\, dx$$

[7]

2 Relative to a fixed origin O, points A and B have coordinates (1, 3, 5) and (–2, 7, 4) respectively.
A laser beam is directed from A towards a hole at B.

(i) Find the distance AB. [2]

The path of the laser beam can be modelled by the equation of a straight line.

(ii) Find the vector equation of this line. [4]

A target is placed at the point with coordinates (16, –17, 9).

(iii) Find whether or not the laser will hit the target. [2]

3 Solve the equation

$$\sin 2\theta = \cot \theta$$

where $-\pi \leqslant \theta \leqslant \pi$ [8]

4 Given the functions

$$f(x) = \frac{5}{x+1} \qquad x \neq -1$$

$$g(x) = 2x + 3 \qquad x \geqslant 0$$

$$h(x) = x^2 \qquad x \geqslant 0$$

(i) Find the composite function $fg(x)$. [2]

(ii) Find the inverse function $f^{-1}(x)$ stating its domain. [5]

(iii) Sketch the graph of $y = h(x)$. [1]

(iv) Express in terms of f, g and/or h:

(a) $x \rightarrow (2x+3)^2$ [1]

(b) $x \rightarrow \sqrt{\frac{5}{x+1}}$ [1]

(c) $x \rightarrow 4x + 9$ [1]

5 **(i)** For the equation

$$x^3 - 3x^2y = 4$$

use implicit differentiation to show that

$$\frac{dy}{dx} = \frac{x - 2y}{x}$$

[5]

(ii) Hence find the stationary point on the curve

$$x^3 - 3x^2y = 4$$

and determine its nature. [9]

6 Given that $\sin(x - \theta) = 3\cos(x + \theta)$

prove that $\tan x \equiv \dfrac{3 + \tan\theta}{1 + 3\tan\theta}$ [7]

7 **Fig. 1** below shows a sketch of the curve

$$y = 1 + \sin 2x$$

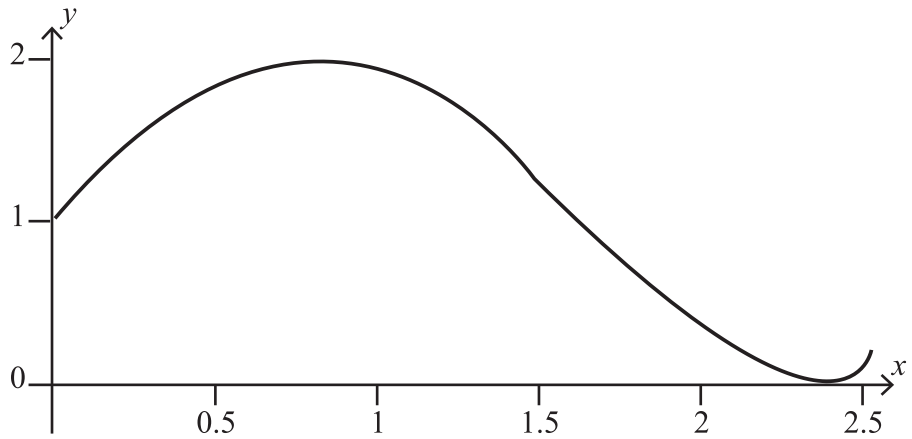

Fig. 1

The curved surface area of a bottle stopper can be modelled by rotating the curve

$$y = 1 + \sin 2x$$

between $x = 0$ and $x = \dfrac{3\pi}{4}$ through 2π radians about the x-axis.

Find the exact volume of the bottle stopper. [10]

8 A curve passes through the point (1, 2) and its gradient function is given by

$$\frac{x \ln x}{e^{2y}}$$

Find the equation of the curve. [10]

THIS IS THE END OF THE QUESTION PAPER