Rewarding Learning

ADVANCED
General Certificate of Education
2016

Mathematics

Assessment Unit F2
assessing
Module FP2: Further Pure Mathematics 2

AMF21

[AMF21]
FRIDAY 17 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(i)** Show that

$$\sum_{r=1}^{n} r(r+1) = \frac{n(n+1)(n+2)}{3}$$ [3]

(ii) Using this result, evaluate

$$\sum_{r=11}^{20} r(r+1)$$ [2]

2 **(i)** Using partial fractions, show that

$$\frac{2x^3 + 6x^2 + 20x + 45}{(x+2)(x^2+9)} \equiv 2 + \frac{1}{x+2} + \frac{x}{x^2+9}$$ [6]

(ii) Hence or otherwise find the exact value of

$$\int_0^1 \frac{2x^3 + 6x^2 + 20x + 45}{(x+2)(x^2+9)}\,dx$$ [5]

3 **(a)** Find, in radians, the general solution of the equation

$$8\sin\theta + 15\cos\theta = 6$$ [6]

(b) Using small angle approximations, show that when x is small

$$\frac{1+\cos x}{1+\sin\left(\frac{x}{2}\right)} \approx 2 - x$$ [3]

4 **(i)** Using Maclaurin's theorem, show that a series expansion for $\ln(1 + 2x)$ up to and including the term in x^4 is

$$\ln(1 + 2x) = 2x - 2x^2 + \frac{8x^3}{3} - 4x^4 \qquad [6]$$

(ii) Hence or otherwise find the corresponding Maclaurin's expansion for

$$\ln\left\{\sqrt[3]{\frac{1 + 2x}{1 - 2x}}\right\} \qquad [3]$$

5 **(i)** Show that the equation of the tangent to the parabola $y^2 = 4ax$ at the point P $(at^2, 2at)$ is

$$ty = x + at^2 \qquad [4]$$

A line perpendicular to this tangent is drawn through the origin.
It intersects the tangent at a point Q.

(ii) As t varies find a Cartesian equation for the locus of Q. [4]

6 Prove using mathematical induction that for $n \geqslant 1$

$$1^2 + 4^2 + 7^2 + \text{................} + (3n - 2)^2 = \frac{n}{2}(6n^2 - 3n - 1) \qquad [7]$$

7 The electric current i (amperes) flowing through a coil of inductance L (henrys) and a resistor of resistance R (ohms) due to an applied voltage E satisfies the differential equation

$$L\frac{di}{dt} + R\,i = E$$

where L, R and E are positive constants and t is the time in seconds.

(i) If $i = 0$ when $t = 0$, find an expression for i at time t. [8]

(ii) Describe what happens to i as t becomes very large. [2]

8 **(i)** Using De Moivre's theorem, show that

$$\sin 5\theta \equiv 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$$ [6]

(ii) Hence show that if θ is not a multiple of π

$$\frac{\sin 5\theta}{\sin \theta} \equiv 16 \cos^4 \theta - 12 \cos^2 \theta + 1$$ [3]

(iii) By solving the equation $\sin 5\theta = 0$, deduce that

$$\cos^2\left(\frac{\pi}{5}\right) = \frac{3+\sqrt{5}}{8}$$ [6]

(iv) Write down the corresponding value for $\cos^2\left(\frac{2\pi}{5}\right)$ [1]

THIS IS THE END OF THE QUESTION PAPER