Rewarding Learning

ADVANCED

General Certificate of Education

2016

Mathematics

Assessment Unit C3

assessing

Module C3: Core Mathematics 3

AMC31

[AMC31]

FRIDAY 20 MAY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

9820.05 **R**

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Solve the equation

$$2\sec^2\theta = 3 + \tan\theta$$

for $0° \leqslant \theta \leqslant 180°$ [6]

2 **(i)** Use the binomial theorem to find, in ascending powers of x, the expansion of

$$(1 + 2x)^{-\frac{1}{2}}$$

as far as the term in x^3 [4]

(ii) State the range of values of x for which the expansion is valid. [1]

3 **(a)** On separate diagrams sketch the graphs of

(i) $y = |x + 2|$ [2]

(ii) $y = e^x + 2$ [2]

(iii) $y = |\ln x|$ [2]

labelling any relevant points.

(b) Write

$$\frac{x^3 + 1}{x^3 - x^2}$$

in partial fractions. [10]

4 **Fig. 1** below shows the logo for an opticians.

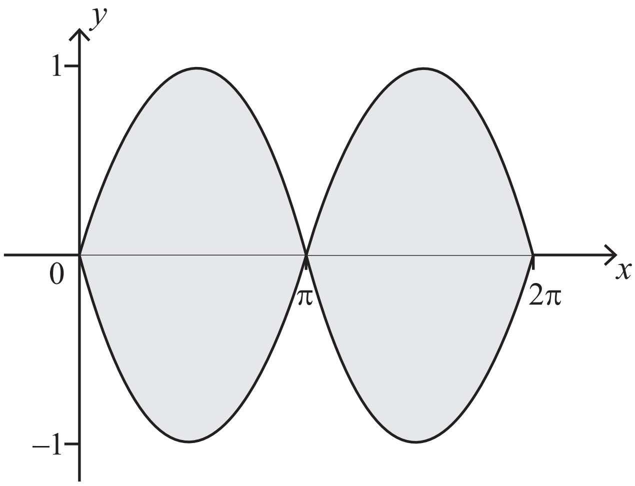

Fig. 1

The area of the logo can be modelled as the area bounded by the curves $y = \sin x$ and $y = -\sin x$ between $x = 0$ and $x = 2\pi$.

Find this area. [5]

5 **(i)** Find the gradient of the curve

$$y = x^2 + x\ln x \qquad x > 0$$

at any point (x, y). [4]

(ii) Show that a stationary point of the curve occurs between $x = 0.2$ and $x = 0.3$ [4]

(iii) By taking $x = 0.2$ as a first approximation to the x coordinate of this stationary point and applying the Newton-Raphson method once, find a better approximation to this coordinate. [4]

6 A sample of radium loses mass at a rate of 4% per century.

Find, in years, the half-life of radium, i.e. the time taken for its mass to be halved. [6]

7 **(i)** Show that

$$(x^2 - x + 2)$$

is positive for all values of x. [4]

(ii) Given that

$$y = (x^2 + 2)e^{-2x}$$

show that $\frac{dy}{dx} < 0$ for all values of x. [7]

8 **(a)** Differentiate with respect to x

(i) $4\sin x - \ln(1 - x^2)$ [3]

(ii) $\frac{\operatorname{cosec}^2 x}{\tan 3x}$ [6]

(b) Show that

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4}{3 - 3\sin^2 2x}\, dx = \frac{-4\sqrt{3}}{3}$$ [5]

* Please see note below regarding question 8 part (b).
This question is designed to assess the application of the techniques of integration to a definite integral involving trigonometric functions. In this case, the integral does not have a finite value due to the presence of a singularity in the integrand within the specified limits.

THIS IS THE END OF THE QUESTION PAPER