Rewarding Learning

ADVANCED
General Certificate of Education
2014

Mathematics

Assessment Unit C3
assessing
Module C3: Core Mathematics 3

[AMC31]

MV18

THURSDAY 15 MAY, AFTERNOON

TIME

1 hour 30 minutes, plus your additional time allowance.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75.
Figures in brackets printed at the end of each question indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

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(Questions start overleaf)

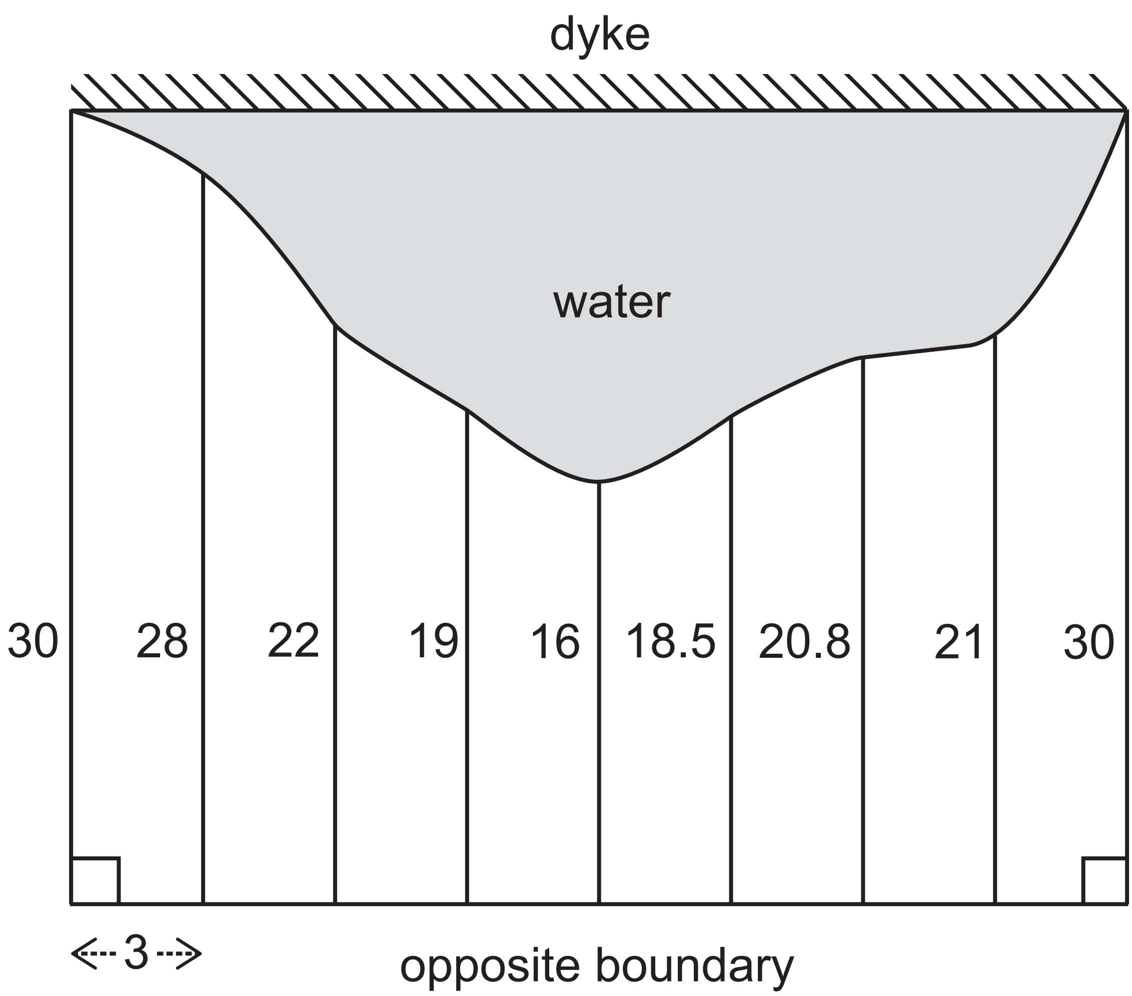
Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Farmer Fred's field floods. The area under water adjoins a dyke. Fred takes measurements, in metres, 3 m apart, from the water's edge to the opposite boundary as shown in **Fig. 1** below:

Fig. 1



Use Simpson's Rule to find an approximation to the surface area of the field which is not under water. [4 marks]

2 A curve is defined parametrically by

$$x = at \qquad y = \frac{a}{1+t^2}$$

Derive a Cartesian equation of this curve. [3 marks]

3 Given the functions:

$$\mathrm{f}(x) = \frac{2x^2 - x - 15}{2x - 5}$$

and $$\mathrm{g}(x) = \frac{x - 3}{4x^2 - 25}$$

write $\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$ in the form $(ax + b)^2$, where a and b are integers. [5 marks]

4 **(a)** **(i)** Express

$$\frac{2}{x^2 - 1}$$

in partial fractions. [5 marks]

(ii) Hence, or otherwise, show that

$$\frac{1}{\sec\theta - 1} - \frac{1}{\sec\theta + 1} \equiv 2\cot^2\theta \qquad [4 \text{ marks}]$$

(b) (i) Expand

$$(1 + 5x)^{\frac{2}{5}}$$

in a binomial series up to and including the term in x^3 [4 marks]

(ii) Hence, **write down** a similar expansion for

$$(1 - 5x)^{\frac{2}{5}}$$ [2 marks]

(iii) Hence, assuming x^4 and higher powers of x can be ignored, solve the equation

$$\sqrt[5]{(1+5x)^2} + \sqrt[5]{(1-5x)^2} = 1\frac{7}{8}$$ [3 marks]

5 **(a)** Find

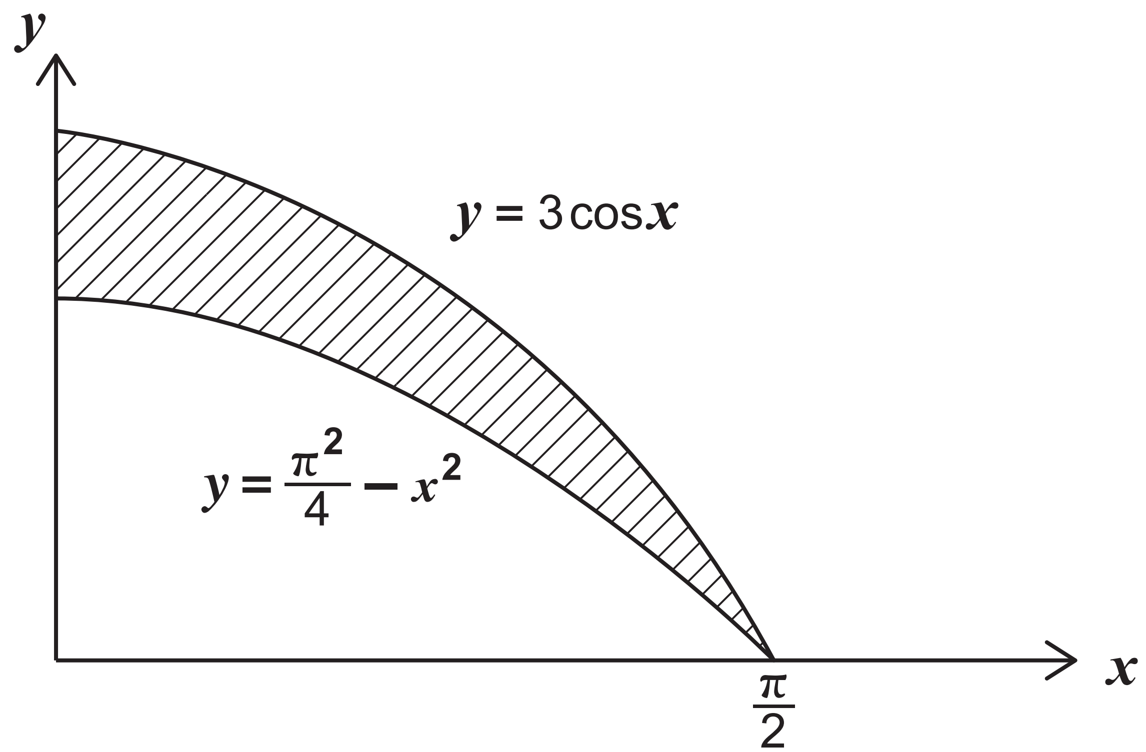
$$\int \left(\operatorname{cosec}^2 2x + \sqrt{x} - \frac{3}{x} + \mathrm{e}^{-3x}\right) \mathrm{d}x \quad \text{[5 marks]}$$

(b) The blade of a knife can be modelled as the area bounded by the curves

$y = 3\cos x$ and $y = \dfrac{\pi^2}{4} - x^2$ and the y-axis,

as shown shaded in **Fig. 2** below.

Fig. 2



Calculate the exact area of the blade. [6 marks]

6 **(a)** **(i)** Sketch the graph of

$$y = |\sin x|$$

for $-360° \leqslant x \leqslant 360°$ [2 marks]

(ii) Write down a function whose graph is shown in **Fig. 3** opposite. [2 marks]

(b) Solve the equation

$$\operatorname{cosec}\left(2\theta + \frac{\pi}{3}\right) = 2$$

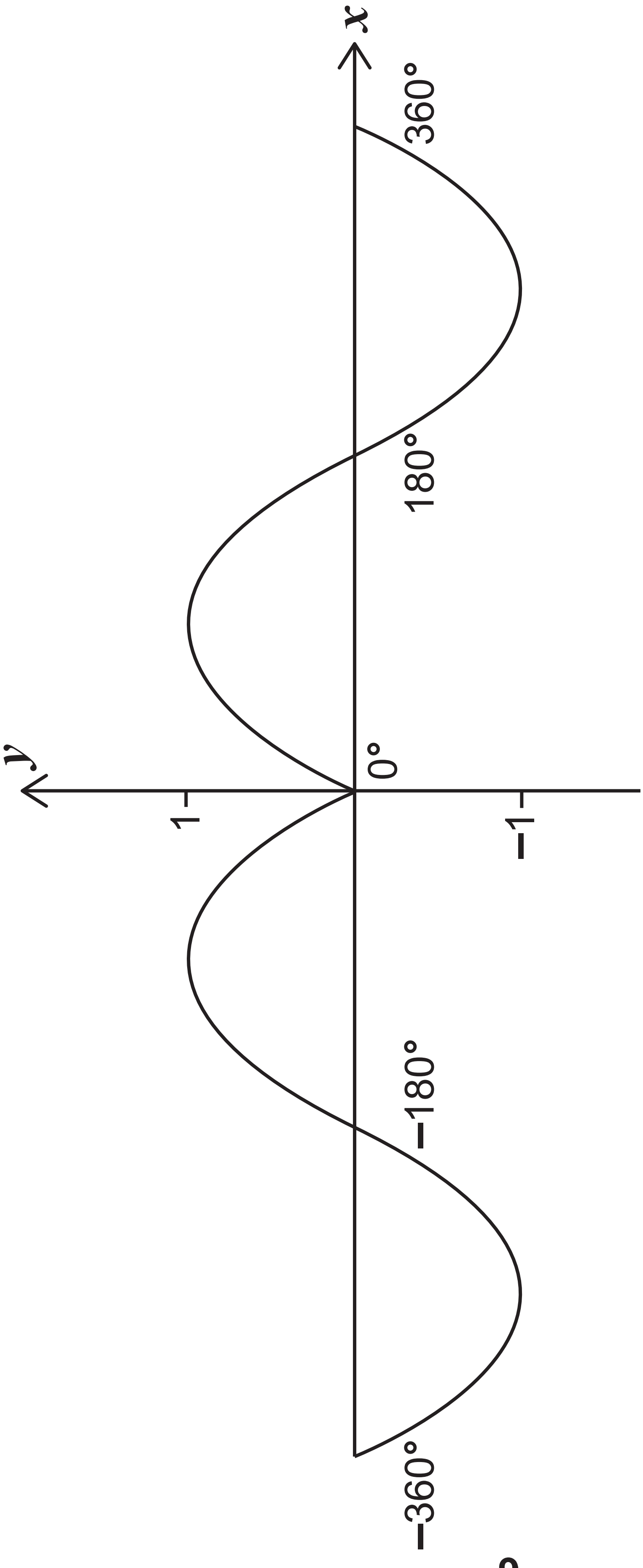
where $-\pi \leqslant \theta \leqslant \pi$ [7 marks]

7 Find the exact equation of the tangent to the curve

$$y = \tan\left(2x - \frac{\pi}{6}\right)\cos\left(x + \frac{\pi}{12}\right)$$

at the point where $x = \frac{\pi}{4}$ [10 marks]

Fig. 3

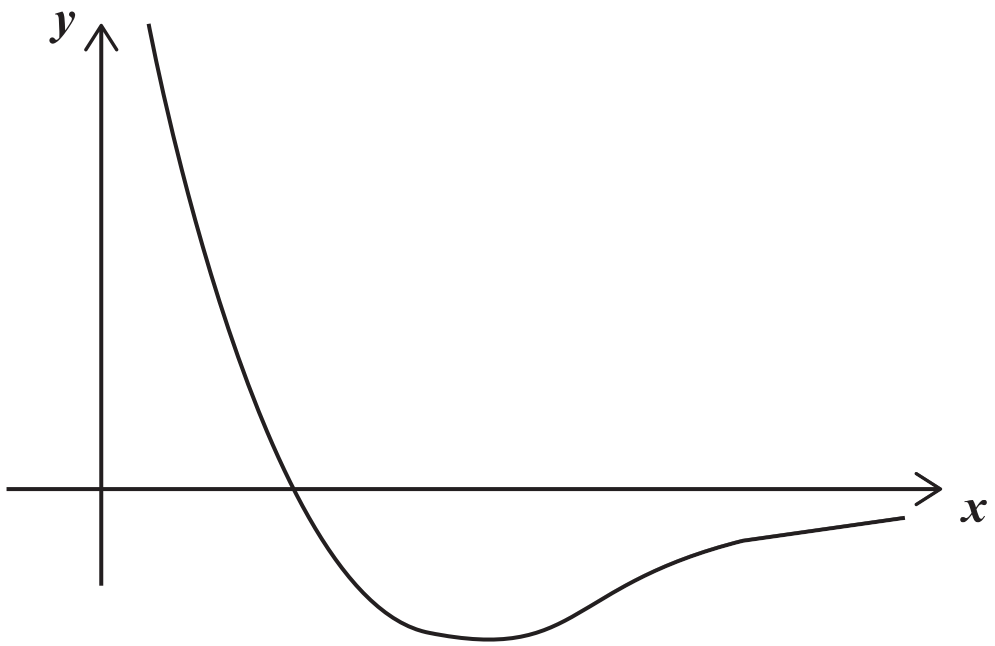


8 A potential well in nuclear physics may be modelled by the function

$$y = -\frac{\ln x}{e^x}$$

shown sketched in **Fig. 4** below:

Fig. 4



(i) Find $\frac{dy}{dx}$ [3 marks]

(ii) Show that the x coordinate of the stationary point on the curve $y = -\frac{\ln x}{e^x}$ satisfies the equation

$\ln x = \frac{1}{x}$ [3 marks]

(iii) Hence, using the Newton–Raphson method twice with starting value 2, find an approximation to the coordinates of the stationary point. [7 marks]

THIS IS THE END OF THE QUESTION PAPER