General Certificate of Secondary Education
2006

## Additional Mathematics

# Paper 1 <br> Pure Mathematics 

[G0301]

## TUESDAY 16 MAY, AFTERNOON

## TIME

2 hours.

## INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet and the Supplementary Answer Booklet provided.
Answer all eleven questions.
At the conclusion of the examination attach the Supplementary Answer Booklet to your Answer Booklet using the treasury tag supplied.

## INFORMATION FOR CANDIDATES

The total mark for this paper is 100 .
Figures in brackets printed down the right-hand side of pages indicate the approximate marks awarded to each question or part question.
You may use your calculator.
A copy of the formulae list is provided.

## Answer all eleven questions.

1 (i) Using the axes and scales in Fig. 1 in your Supplementary Answer Booklet, sketch the graph of $y=\sin x$ for $0^{\circ} \leqslant x \leqslant 360^{\circ}$
(ii) Hence, using the axes and scales in Fig. 2 in your Supplementary Answer Booklet, sketch the graph of $y=\sin x-1$ for $0^{\circ} \leqslant x \leqslant 360^{\circ}$

2 Solve the equation

$$
\sin \left(\frac{1}{2} x+80^{\circ}\right)=0.6
$$

for $-180^{\circ}<x \leqslant 180^{\circ}$
Give your answers correct to 2 decimal places.

3 (i) Find $\mathbf{A}^{-1}$ where $\mathbf{A}=\left[\begin{array}{ll}3 & 8 \\ 2 & 6\end{array}\right]$
(ii) Hence, using a matrix method, solve the following simultaneous equations for $x$ and $y$.

$$
\begin{align*}
& 3 x+8 y=13 \\
& 2 x+6 y=9 \tag{4}
\end{align*}
$$

4 (a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$ when $y=2 x^{7}-\frac{5}{x^{4}}-7$
(b) Find $\int\left(\frac{2}{x^{7}}-5 x^{4}\right) \mathrm{d} x$

5 (i) Show that

$$
\frac{3 x-1}{x+2}+\frac{2 x-1}{x+4}
$$

can be written as

$$
\begin{equation*}
\frac{5 x^{2}+14 x-6}{x^{2}+6 x+8} \tag{4}
\end{equation*}
$$

(ii) Hence, or otherwise, solve the equation

$$
\begin{equation*}
\frac{3 x-1}{x+2}+\frac{2 x-1}{x+4}=3 \tag{4}
\end{equation*}
$$

6 (a) If $\log _{4} 64=a$ what is the value of $a$ ?
(b) If $\log _{3} 2=b$ express $\log _{3} 18$ in terms of $b$.
(c) Solve the equation

$$
5^{\left(1+\frac{1}{2} x\right)}=21
$$

giving your answer correct to 3 decimal places.

7 Fig. 3 shows a sketch of the graph of the function $y=x^{2}-4 x+7$


Fig. 3
(i) Find the equation of the tangent at the point $\mathrm{A}(1,4)$.
(ii) Find the equation of the tangent to the curve which has a gradient of 6
(iii) Verify that both tangents meet at the point $(3,0)$.

8 The points $\mathrm{A}, \mathrm{O}$ and B lie in a straight line on horizontal ground, with $\mathrm{AO}=1200 \mathrm{~m}$ and $\mathrm{OB}=500 \mathrm{~m}$, as shown in Fig. 4.


Fig. 4

A hot-air balloon takes off from A and rises at a constant angle of $15.0^{\circ}$ to the horizontal in a plane vertically above the line AOB. When it is at the point X its angle of elevation to an observer at O is $65.0^{\circ}$
(i) Calculate the distance OX.

The balloon then descends along the path XB, reaching the ground at B, as shown in Fig. 4.
(ii) Calculate the distance BX .
(iii) Calculate the angle to the horizontal at which the balloon descends.

9 With her digital camera Ann can take high, medium and low resolution photos.
One day she took 10 high, 15 medium and 20 low resolution photos and she used 45 Mb (megabytes) of storage on her memory stick.

Let $x, y$ and $z$ represent the amounts of storage, in megabytes, used by a high, a medium and a low resolution photograph respectively.
(i) Show that $x, y$ and $z$ satisfy the equation

$$
\begin{equation*}
2 x+3 y+4 z=9 \tag{1}
\end{equation*}
$$

On the next day Ann took 14 high, 12 medium and 20 low resolution photographs which used 50 Mb of storage.
(ii) Show that $x, y$ and $z$ also satisfy the equation

$$
\begin{equation*}
7 x+6 y+10 z=25 \tag{1}
\end{equation*}
$$

On the third day Ann took 15 high, 12 medium and 36 low resolution photographs which used 60 Mb of storage.
(iii) Show that $x, y$ and $z$ also satisfy the equation

$$
\begin{equation*}
5 x+4 y+12 z=20 \tag{1}
\end{equation*}
$$

(iv) Solve these equations, showing clearly each stage of your solution.

The camera has a video mode. Six seconds of video recording uses the same amount of storage on the memory stick as a high resolution photograph.

The total amount of storage on the memory stick is 256 Mb .
(v) Show that, in addition to all the photographs taken so far, Ann could get just over 5 minutes of video before the memory stick would be full.

10 At an Arctic weather station the wind chill index $W$ was recorded for various wind speeds $V$ and the results are given in Table 1.

Table 1

| Wind speed <br> $V(\mathrm{~km} / \mathrm{h})$ | Wind chill index <br> $W$ |
| :---: | :---: |
| 5 | 25.5 |
| 10 | 28.5 |
| 25 | 33.0 |
| 50 | 36.9 |
| 80 | 39.8 |

It is believed that a relationship of the form

$$
W=k V^{n}
$$

exists between $W$ and $V$, where $k$ and $n$ are constants.
(i) Using Fig. 5 in your Supplementary Answer Booklet verify this relationship by drawing a suitable straight line graph, using values correct to three decimal places. Label the axes clearly.
(ii) Hence, or otherwise, obtain values for $k$ and $n$.
(iii) Use the formula $W=k V^{n}$ with the values you obtained for $k$ and $n$ to calculate the wind chill index when the wind speed is $100 \mathrm{~km} / \mathrm{h}$. State any assumption which you make. [3]

When the wind chill index reaches 35 there is a high risk of frostbite.
(iv) Use the formula $W=k V^{n}$ to determine the wind speed which would give a wind chill index of 35

11 A curve is defined by the equation

$$
y=x(x+1)(x-2)
$$

(i) Write down the coordinates of the points where the curve crosses the $x$-axis.
(ii) Find, correct to 2 decimal places, the coordinates of the turning points and identify each as either a maximum or a minimum point.
(iii) Sketch the curve using Fig. 6 in your Supplementary Answer Booklet.
(iv) Find the area above the $x$-axis which is enclosed by the curve and the $x$-axis.

