# GCSE

## Additional Mathematics

## Summer 2008

# Mark Schemes

Issued: October 2008

**NORTHERN IRELAND GENERAL CERTIFICATE OF SECONDARY EDUCATION (GCSE)**
**AND NORTHERN IRELAND GENERAL CERTIFICATE OF EDUCATION (GCE)**

**MARK SCHEMES (2008)**

## Foreword

### *Introduction*

Mark Schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

### *The Purpose of Mark Schemes*

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of 16- and 18-year-old students in schools and colleges. The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes therefore are regarded as a part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

The Council hopes that the mark schemes will be viewed and used in a constructive way as a further support to the teaching and learning processes.

**CONTENTS**

Rewarding Learning

General Certificate of Secondary Education
2008

# Additional Mathematics

Paper 1
Pure Mathematics

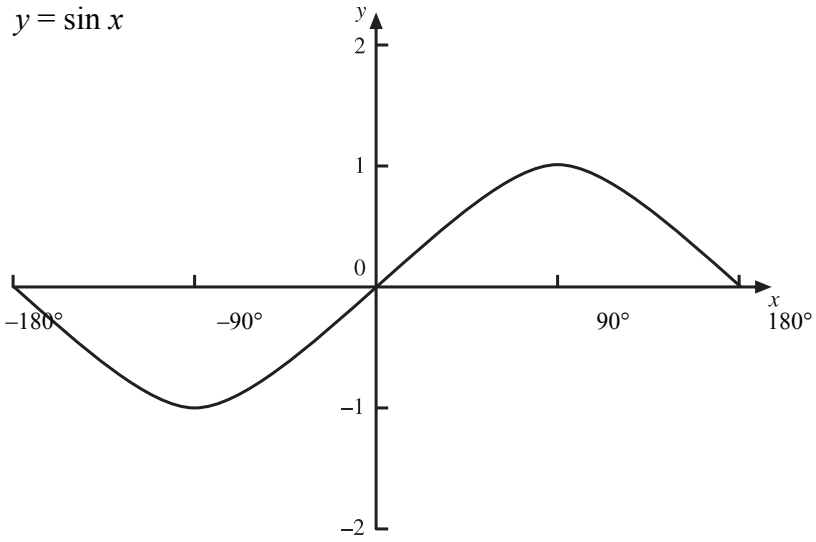
**[G0301]**

**TUESDAY 13 MAY, AFTERNOON**

# MARK SCHEME

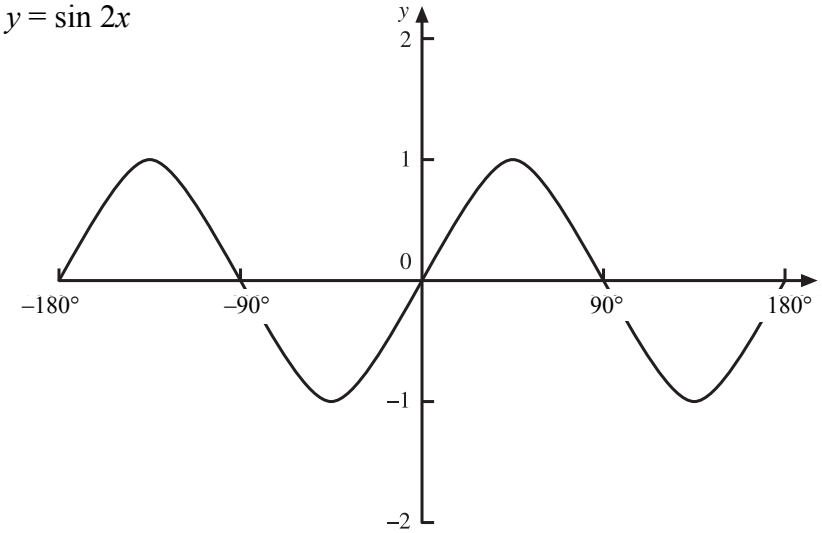
| | AVAILABLE MARKS |
|---|---|

**1** **(i)** $y = \sin x$




M1 (shape)
W1 (axes)

**(ii)** $y = \sin 2x$




M1 (shape)
W1 (axes)

4

**2** **(i)** $\tan\theta = 2$

$\therefore \theta = 63.43°$ or $243.43°$ to 2 d.p. 2 × MW1

**(ii)** $\tan(4x + 5°) = 2$

From **(i)**

$4x + 5 = 63.43$ or $243.43$ M1

$4x = 58.43$ or $238.43$

$x = 14.61$ or $59.61$ to 2 d.p. 2 × MW1

5

| | | | AVAILABLE MARKS |
|---|---|---|---|
| **3** | **(i)** $\det A = 3$ | MW1 | |
| | $A^{-1} = \frac{1}{3}\begin{bmatrix} 6 & 3 \\ 7 & 4 \end{bmatrix}$ | MW1 | |
| | **(ii)** $\begin{bmatrix} 4 & -3 \\ -7 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ -8 \end{bmatrix}$ | M1 | |
| | $\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -7 & 6 \end{bmatrix}^{-1}\begin{bmatrix} -5 \\ -8 \end{bmatrix}$ | | |
| | $= \frac{1}{3}\begin{bmatrix} 6 & 3 \\ 7 & 4 \end{bmatrix}\begin{bmatrix} -5 \\ -8 \end{bmatrix}$ | M2 | |
| | $= \frac{1}{3}\begin{bmatrix} -6 \\ -3 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$ | | |
| | $\therefore x = -2,\ y = -1$ | W1 | 6 |
| **4** | **(a)** $y = 3x^4 + \frac{4}{x^5} - 1$ | | |
| | $\therefore \frac{dy}{dx} = 12x^3 - \frac{20}{x^6}$ | 2 × MW1 | |
| | **(b)** $\int\left(3x^4 + \frac{4}{x^5} - 1\right)dx$ | | |
| | $= \frac{3}{5}x^5 - \frac{1}{x^4} - x + c$ | 4 × MW1 | 6 |

| | | | AVAILABLE MARKS |
|---|---|---|---|
| **5** | **(i)** $y = -x^3 + 6x^2 - 5x$ | | |
| | $\therefore \frac{dy}{dx} = -3x^2 + 12x - 5$ | M1, W1 | |
| | At $x = 1$, $\frac{dy}{dx} = 4$ | W1 | |
| | $\therefore$ Equation of tangent is | | |
| | $y = 4(x - 1)$ | | |
| | i.e. $y = 4x - 4$ | MW1 | |
| | **(ii)** At B, $\frac{dy}{dx} = 4$ | | |
| | $\therefore -3x^2 + 12x - 5 = 4$ | M1 | |
| | $\therefore -3x^2 + 12x - 9 = 0$ | | |
| | $\therefore \quad x^2 - 4x + 3 = 0$ | W1 | |
| | $\therefore \quad (x - 3)(x - 1) = 0$ | | |
| | $\therefore \quad x = 1$ (point A) or $x = 3$ (point B) | W1 | |
| | When $x = 3$, $y = 12$ | | |
| | So B is the point (3, 12) | W1 | 8 |
| **6** | **(i)** $\frac{2x-1}{3x+4} - \frac{2x+1}{4-x}$ | | |
| | $= \frac{(2x-1)(4-x) - (2x+1)(3x+4)}{(3x+4)(4-x)}$ | M2 | |
| | $= \frac{(-2x^2 + 9x - 4) - (6x^2 + 11x + 4)}{-3x^2 + 8x + 16}$ | | |
| | $= \frac{-8x^2 - 2x - 8}{-3x^2 + 8x + 16}$ | W1 (numerator)<br>W1 (denominator) | |
| | $= \frac{2(4x^2 + x + 4)}{3x^2 - 8x - 16}$ | | |
| | **(ii)** $\frac{2(4x^2 + x + 4)}{3x^2 - 8x - 16} = 2$ | | |
| | $\therefore 2(4x^2 + x + 4) = 2(3x^2 - 8x - 16)$ | M2 | |
| | $\therefore \quad 4x^2 + x + 4 = 3x^2 - 8x - 16$ | | |
| | $\therefore \quad x^2 + 9x + 20 = 0$ | W1 | |
| | $\therefore \quad (x + 4)(x + 5) = 0$ | | |
| | $\therefore \quad x = -4$ or $x = -5$ | W1 | 8 |

**7** **(a)** $\log_a 32 = 5$

$\therefore \quad a^5 = 32$

$\therefore \quad a = 2$ MW1

**(b)** $\log_2 50 = \log_2 (25 \times 2)$ M1

$= \log_2 5^2 + \log_2 2$ M1

$= 2\log_2 5 + \log_2 2$

$= 2b + 1$ MW1

**(c)** $3^{\left(7-\frac{x}{2}\right)} = 8$

$\therefore \left(7 - \frac{x}{2}\right)\log 3 = \log 8$ M2

$\therefore 7 - \frac{x}{2} = \frac{\log 8}{\log 3}$ M1

$\therefore x = 2\left\{7 - \frac{\log 8}{\log 3}\right\}$

$= 10.21$ to 2 d.p. W1 8

AVAILABLE MARKS

**8** **(i)** $AB^2 = AY^2 + BY^2 - 2 \times AY \times BY \cos A\hat{Y}B$

$= 22^2 + 18.5^2 - 2 \times 22 \times 18.5 \cos 56.45$ M2

$\therefore AB = 19.40\,\text{km}.$ W1

**(ii)** $\dfrac{\sin B\hat{A}Y}{BY} = \dfrac{\sin A\hat{Y}B}{AB}$

$\therefore \sin B\hat{A}Y = \dfrac{18.5 \sin 56.45}{19.4}$ M2

$\therefore \quad B\hat{A}Y = 52.63°$ W1

**(iii)** $A\hat{B}Y = 180 - 56.45 - 52.63$

$= 70.92°$ MW1

**(iv)** $\tan X\hat{B}Y = \dfrac{XY}{BY} = \dfrac{6.11}{18.5}$ M1

$\therefore \quad X\hat{B}Y = 18.28°$ W1

**(v)** $A\hat{B}X = A\hat{B}Y - X\hat{B}Y$

$= 70.92 - 18.28$

$= 52.64°$

$\simeq B\hat{A}Y$ W1

**(vi)** As $O\hat{A}B \simeq O\hat{B}A = 52.64°$,

triangle ABO is isosceles.

So AO $\simeq$ BO, and as boats are travelling at same speed they meet at O. MW1

AVAILABLE MARKS

11

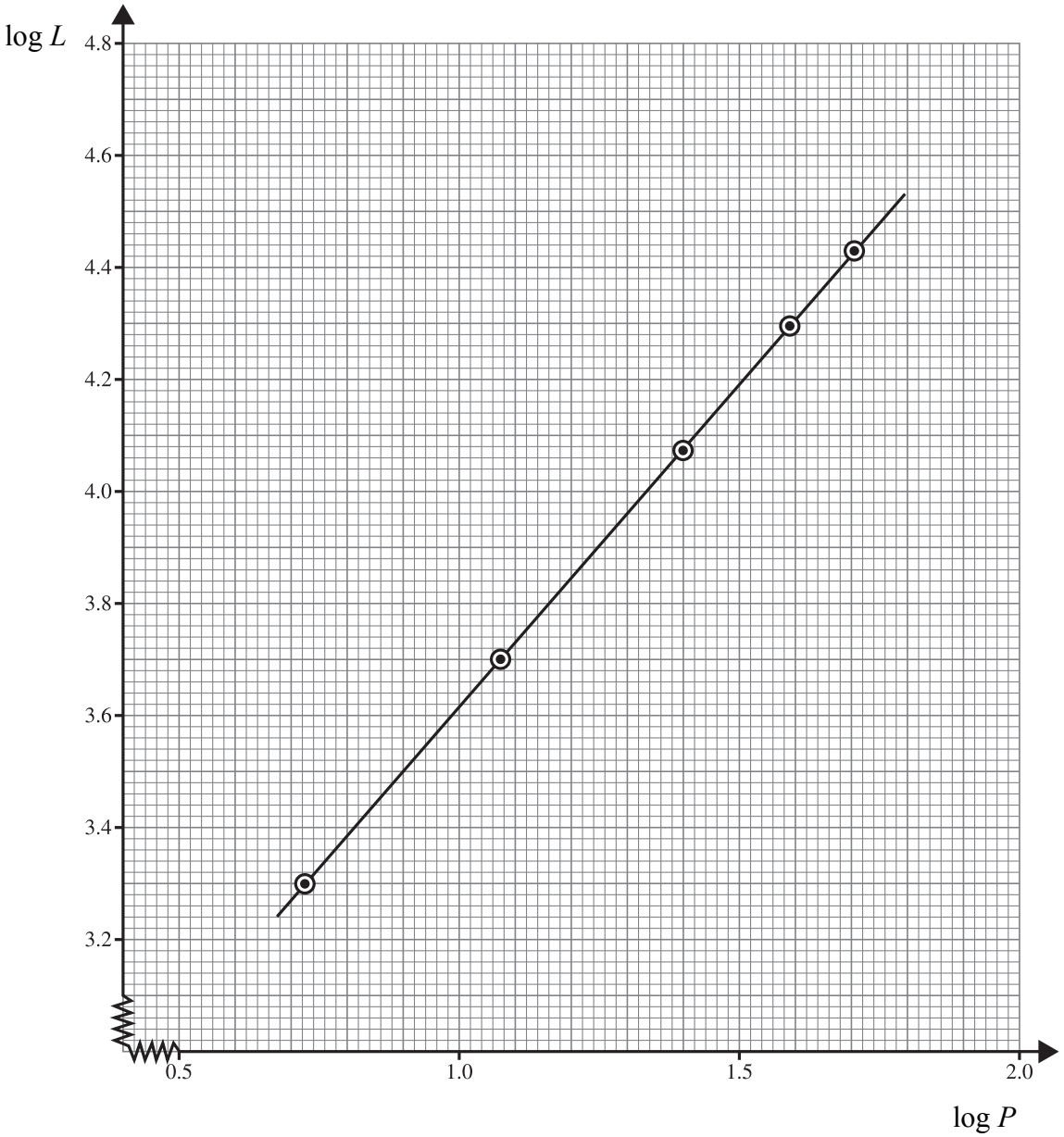
AVAILABLE MARKS

**9 (i)** $L = kP^n$

$\therefore \log L = n \log P + \log k.$ M1

| log $P$ | log $L$ |
|---|---|
| 0.73 | 3.30 |
| 1.06 | 3.69 |
| 1.40 | 4.08 |
| 1.59 | 4.29 |
| 1.71 | 4.44 |

W2




Graph is a straight line so formula holds.

W1 (labels)

W1 (points)

W1 (line)

**(ii)** $n = \text{gradient} = \dfrac{4.44 - 3.30}{1.71 - 0.73} = 1.16$ M1, W1

$k = \dfrac{P}{V^n} = \dfrac{27390}{51.42^{1.16}} = 284$ M1, W1

So $L = 284P^{1.16}$

**(iii)** $L = 284P^{1.16}$

$= 284 \times 16.39^{1.16}$

$= 7280$ suns (to 3 s.f.) MW1

**(iv)** $L = 284P^{1.16}$

So $P = \left(\dfrac{L}{284}\right)^{\frac{1}{1.16}} = \left(\dfrac{1394}{284}\right)^{\frac{1}{1.16}}$ M1

$= 3.94$ days. W1

Assume that the formula holds beyond the range of the given values. M1 14

AVAILABLE MARKS

**10** **(i)** $\frac{6}{10}x+\frac{4}{10}y+\frac{4}{10}z=40$ MW1

$\therefore \quad 6x+4y+4z=400$

$\therefore \quad 3x+2y+2z=200$ (1) W1

**(ii)** $\frac{4}{10}x+\frac{3}{10}y+\frac{5}{10}z=35$

$\therefore \quad 4x+3y+5z=350$ (2) MW1

**(iii)** $\frac{5}{10}x+\frac{5}{10}y+\frac{3}{10}z=39$

$\therefore \quad 5x+5y+3z=390$ (3) MW1

**(iv)** (2)$\times 3-$ (1)$\times 4 \rightarrow y+7z=250$ (4) M1, W1

(3)$\times 3-$ (1)$\times 5 \rightarrow 5y-z=170$ (5) M1, W1

(4)$\times 5-$ (5) $\rightarrow \quad 36z=1080$ M2

$\therefore \quad z=30$

$\therefore \quad y=250-7z=40$

$\therefore \quad x=\frac{200-2y-2z}{3}=20$ M1, W1

So asking prices are:

| | |
|---|---|
| sunglasses | 20 euro |
| bag | 40 euro |
| watch | 30 euro |

**(v)** For sunglasses and watch, amounts spent are:

| | |
|---|---|
| Becky | 12 + 12 = 24 euro |
| Catriona | 8 + 15 = 23 euro |
| Grace | 10 + 9 = 19 euro |

M1, W1

So Laura spent the least amount by asking Grace. 14

AVAILABLE MARKS

**11 (i)** $y = x^3 - 8x^2 + 16x$

$= x(x^2 - 8x + 16)$

$= x(x-4)^2$ MW1

When $y = 0$, $x = 0$ or $x = 4$ and $x = 4$

$\therefore$ meets $x$-axis at (0, 0) and (4, 0). W1

**(ii)** $\frac{dy}{dx} = 3x^2 - 16x + 16 = 0$ MW1, M1

$\therefore (3x-4)(x-4) = 0$

$\therefore x = \frac{4}{3}$ or $x = 4$ 2 × MW1

When $x = \frac{4}{3} (= 1.33)$, $y = 9.48$

When $x = 4$, $y = 0$

$\frac{d^2y}{dx^2} = 6x - 16$ MW1

When $x = \frac{4}{3}$, $\frac{d^2y}{dx^2} = -8 < 0$ $\therefore$ max

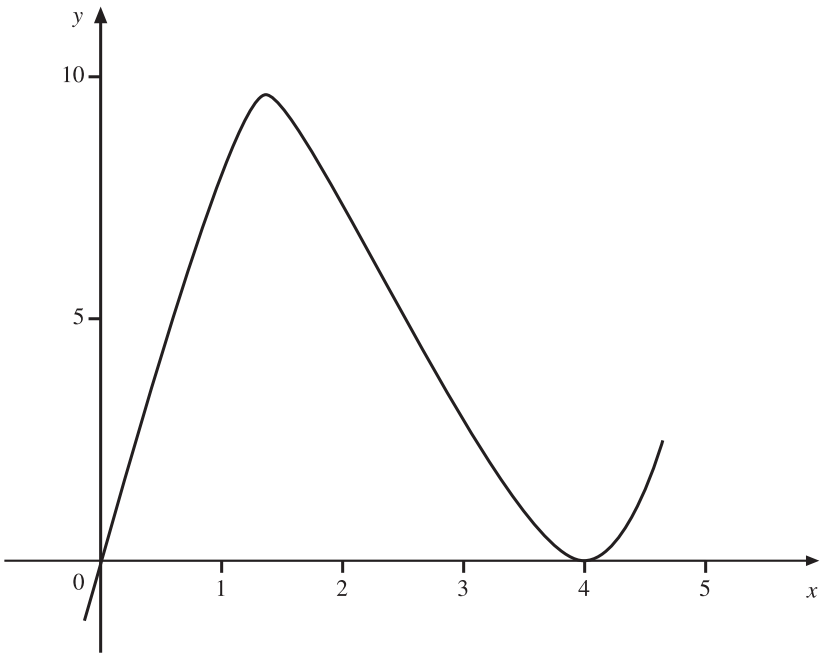
M1

When $x = 4$, $\frac{d^2y}{dx^2} = 8 > 0$ $\therefore$ min

So minimum at (4, 0) W1

So maximum at (1.33, 9.48) (to 2 d.p.) W1

**(iii)**




W1 (shape)

W1 (max)

W1 (min)

AVAILABLE MARKS

**(iv)** $\text{Area} = \int_0^4 (x^3 - 8x^2 + 16x)\,dx$ M1

$= \left[\frac{x^4}{4} - \frac{8x^3}{3} + 8x^2\right]_0^4$ MW1

$= 21.33$ W1

| AVAILABLE MARKS |
|---|
| |
| 16 |

**Total** **100**

Rewarding Learning

General Certificate of Secondary Education
2008

# Additional Mathematics

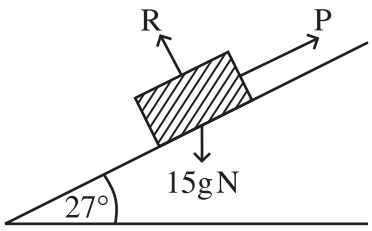
Paper 2
Mechanics and Statistics

**[G0302]**

**THURSDAY 15 MAY, MORNING**

# MARK SCHEME

**1** **(i)**




W1

**(ii)** $R = 15g\cos 27°$

$\Rightarrow R = 133.65 = \underline{133.7\,N}$ MW1

**(iii)** Least force required is P

$P = 15g\sin 27°$ M1

$\Rightarrow P = 68.09 = \underline{68.1N}$ to 1d.p. W1

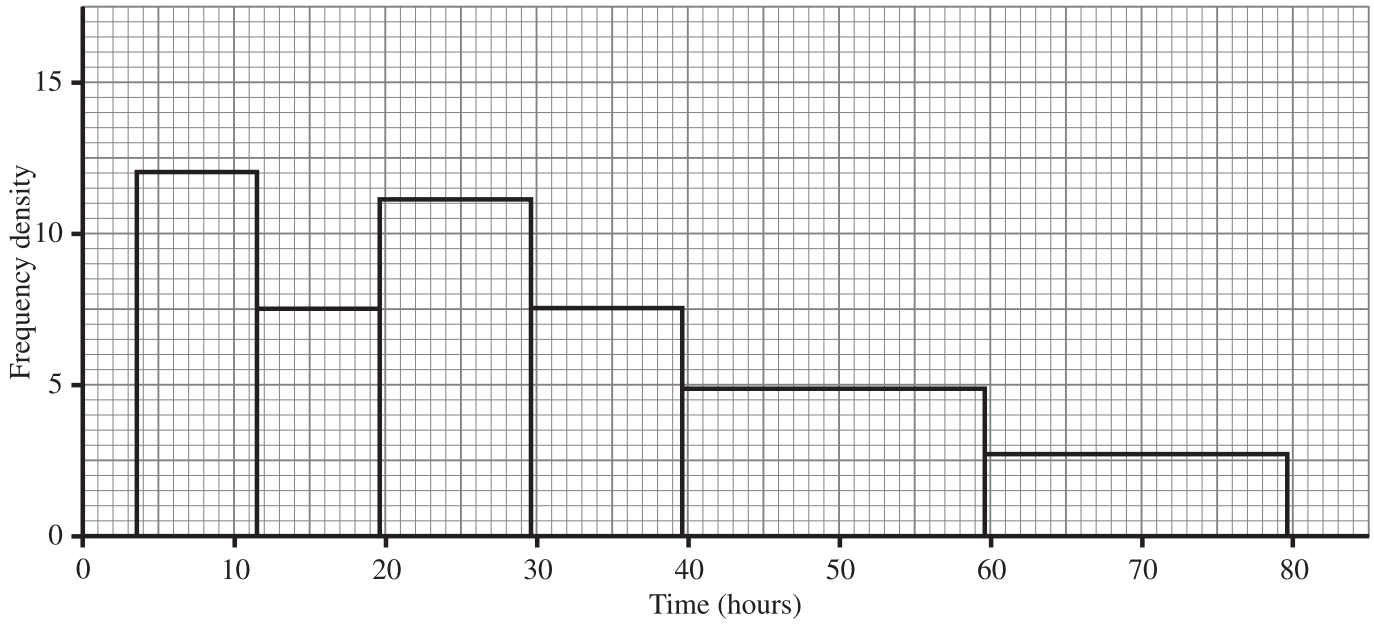
AVAILABLE MARKS

4

**2**

| class width | 8 | 8 | 10 | 10 | 20 | 20 |
|---|---|---|---|---|---|---|
| f.d. | 12 | 7.5 | 11.2 | 7.5 | 4.9 | 2.7 |

M1W1




AVAILABLE MARKS

axes W1
cts scale at boundaries W1
heights W1

5

**3** **(i)** $v^2 = u^2 + 2as$ $u = 0$

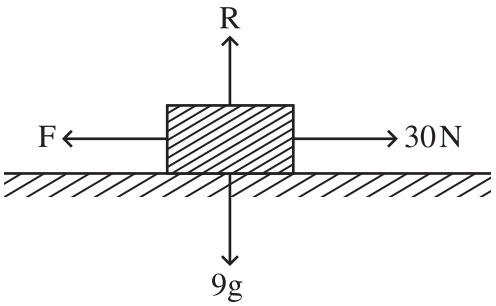
$v = 4\,\text{m/s}$

$s = 24\,\text{m}$

$16 = 0 + 2(a)(24)$

$\Rightarrow a = \frac{16}{48} = \frac{1}{3}\,\text{m/s}^2$ MW1

**(ii)**




accelerating force = mass × acceleration M1

$30 - F = 9 \times \frac{1}{3}$ W1

$\Rightarrow \quad F = 30 - 3 = 27\,\text{N}$

**(iii)** $F = \mu R \qquad R = 9g = 90\,\text{N}$

$\Rightarrow 27 = u.90$ M1

$\Rightarrow \mu = \frac{27}{90}$

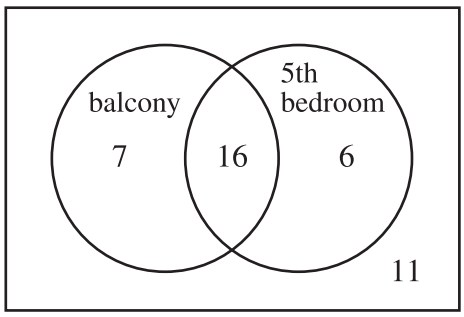
$\Rightarrow \mu = 0.3$ W1 5

**4** Total = 253 median class 40 – 49 MW1

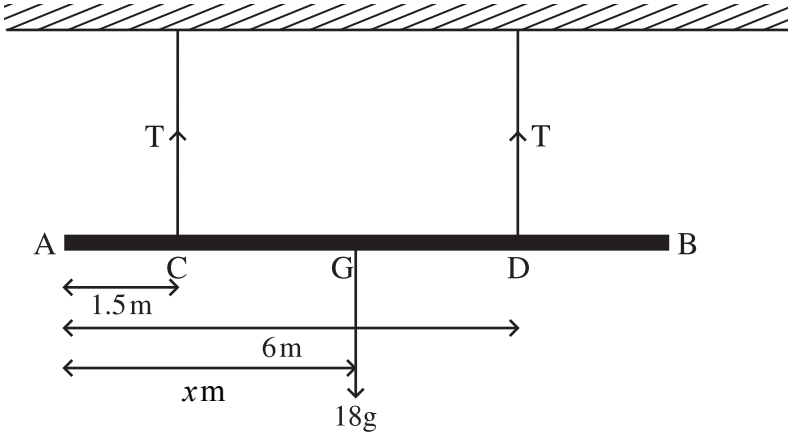
$39.5 + \frac{127 - 97}{60} \times 10$ MW3

$= 44.5$ MW1 5

AVAILABLE MARKS

| | | | AVAILABLE MARKS |
|---|---|---|---|
| **5** | **(i)** $\mathbf{F} = m\mathbf{a}$ | | |
| | $5\mathbf{i} - 12\mathbf{j} = 5\mathbf{a}$ | M1 | |
| | $\Rightarrow \mathbf{a} = (\mathbf{i} - 2.4\mathbf{j})\,\text{m/s}^2$ | W1 | |
| | **(ii)** $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ | | |
| | $\mathbf{s} = 0 + \frac{1}{2}(\mathbf{i} - 2.4\mathbf{j})\,.\,9$ | M1 | |
| | $\Rightarrow \mathbf{s} = (4.5\mathbf{i} - 10.8\mathbf{j})\,\text{m}$ | | |
| | $\Rightarrow \mathbf{OA} = (4.5\mathbf{i} - 10.8\mathbf{j})\,\text{m}$ | W1 | |
| | **(iii)** $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$ | | |
| | $\Rightarrow 5\mathbf{i} - 12\mathbf{j} = (4\mathbf{i} - 7\mathbf{j}) + \mathbf{F}_2$ | M1 | |
| | $\Rightarrow \quad \mathbf{F}_2 = (5\mathbf{i} - 12\mathbf{j}) - (4\mathbf{i} - 7\mathbf{j})$ | | |
| | $\Rightarrow \quad \mathbf{F}_2 = (\mathbf{i} - 5\mathbf{j})\,\text{N}$ | W1 | 6 |
| **6** | **(i)** Overall mean $= \dfrac{54 \times 6 + 64 \times 4}{10} = 58$ | M1W1 | |
| | **(ii)** $18.2^2 = \dfrac{n^2}{6} - 54^2$ $\qquad$ $18.2^2 = \dfrac{p^2}{4} - 64^2$ | M2 | |
| | overall standard deviation $= \sqrt{\left(\dfrac{p^2 + n^2}{10} - 58^2\right)}$ | | |
| | $= 18.8$ | M1W1 | 6 |
| **7** | **(i)**   | MW1 diagram with labels<br>MW1 for 6 and 7<br>MW1 for 11 and 16 | |
| | **(ii)** Using Venn diagram or otherwise P(5th) $= \dfrac{16 + 6}{40} = \dfrac{11}{20}$ | MW1, MW1 | |
| | **(iii)** P(5th bedroom \| no balcony) $= \dfrac{6}{17}$ | MW2 both correct | 7 |

**8**




**(i)** $T + T = 18g$

$\Rightarrow T = 9g = 90\,N$ MW1

**(ii)** Take moments at A **or** Take moments at C

$T(1.5) + T(6) = 18g(x)$ $18g(x - 1.5) = T(6 - 1.5)$ MW3

$\Rightarrow 7.5(90) = 180x$ $18g(x - 1.5) = 9g(4.5)$

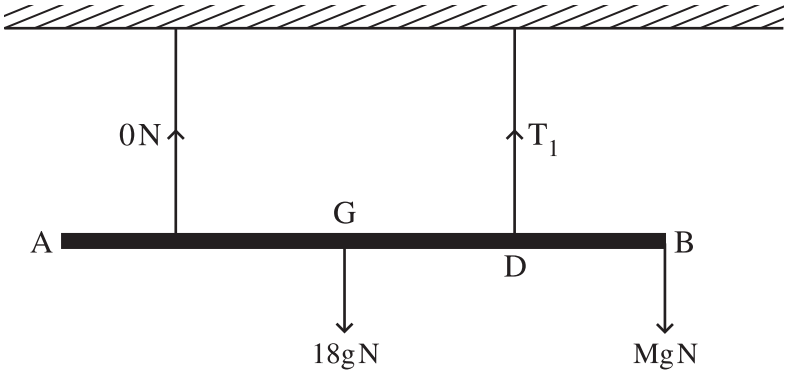
$\Rightarrow x = \dfrac{7.5 \times 90}{180}$ $\Rightarrow 2x - 3 = 4.5$ MW1

$\Rightarrow 2x = 7.5$

$\Rightarrow x = 3.75\,m$ $\Rightarrow x = 3.75\,m$ W1

**(iii)** The package should be suspended at B. M1

**(iv)**




Take moments at D

$Mg.2 = 18g(6 - 3.75)$ MW1, MW1

$\Rightarrow 2M = 18 \times 2.25$

$\Rightarrow 2M = 40.5$

$\Rightarrow M = 20.25\,kg$ W1 10

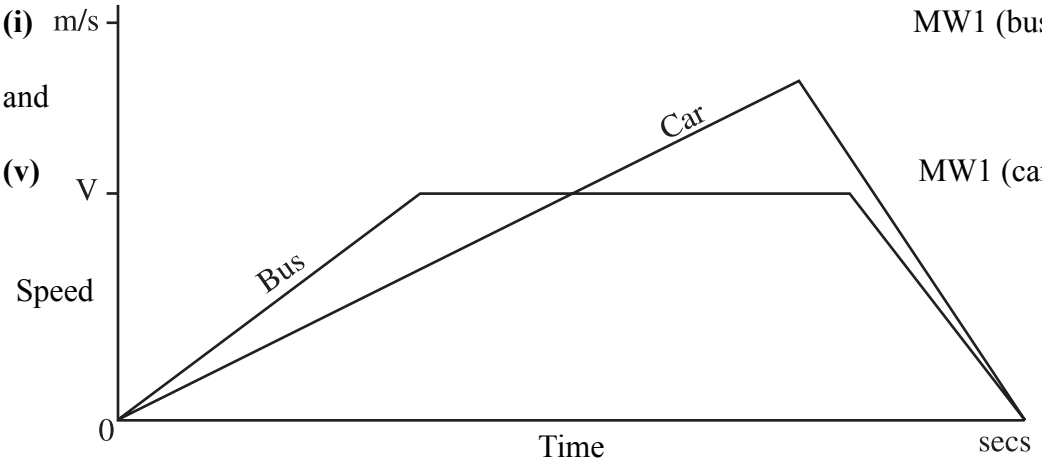
AVAILABLE MARKS

| | AVAILABLE MARKS |
|---|---|

**9** **(i)** m/s — MW1 (bus)

and

**(v)** — MW1 (car)




**(ii)** Using $v = u + at$

$V = 0 + 1.25(16)$

$\Rightarrow V = 20\,\text{m/s}$ MW1

**(iii)** Let $t_1$ = time for bus to reach max speed = 16 secs

$t_2$ = „ „ „ at max speed = 24 secs

$t_3$ = „ „ „ to come to rest

$0 = 20 - 2.5t_3$

$\Rightarrow t_3 = \dfrac{20}{2.5} = 8$ secs MW1

Total time from X to Y = $t_1 + t_2 + t_3$

Time = 16 + 24 + 8 = 48 secs W1

**(iv)** $s_1$ = distance to reach maximum speed

$s_1 = \frac{1}{2}(1.25)16^2 = 160\,\text{m}$

$s_2$ = distance at maximum speed $= 20 \times 24$ MW1

$= 480\,\text{m}$

$s_3$ = distance to come to rest

$s_1 = (20 \times 8) - \frac{1}{2}(2.5)8^2 = 160 - 80 = 80\,\text{m}$ MW1

Total distance = 160 + 480 + 80 = 720 m W1

**or** **or**

From the graph $\frac{1}{2}(48 + 24)20 = 720\,\text{m}$ M1M1W1

**(v)** See diagram above

**(vi)** Car accelerates for $\frac{3}{4}$ of total time $= \frac{3}{4} \times 48 = 36$ secs MW1

**(vii)** Car travels $\frac{3}{4} \times 720 = 540\,\text{m}$ while accelerating M1

Using $s = \dfrac{u + v}{2}t$

$\Rightarrow 540 = \dfrac{0 + v}{2} \cdot 36$ M1

$\Rightarrow v = 30\,\text{m/s}$ W1

12

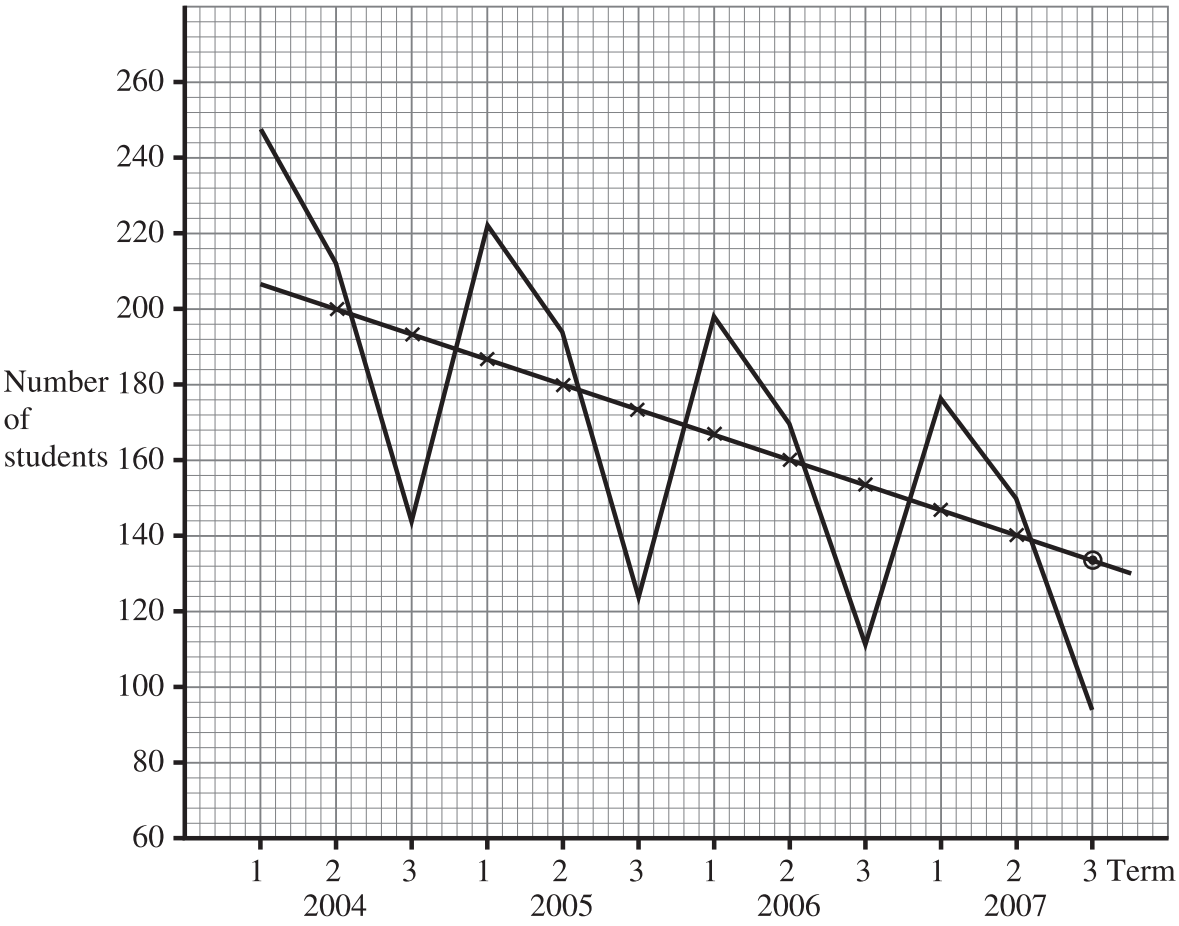
| AVAILABLE MARKS |
|---|

**10** **(i)** 3 point moving averages

200
192
186
180
172
164
160
153
146
140

M2W1

**(ii)**




M2
line MW1

**(iii)** $\frac{x + 94 + 149}{3} = 134$ reading M1W1

$x = 159$ M1W1

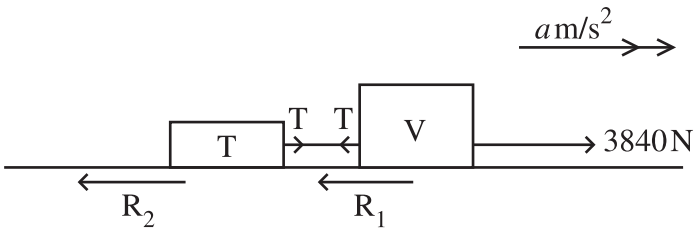
**(iv)** Income from students $= 159 \times 55 = £8745$ MW1

profit $= £3745$

$\%\ \text{profit} = \frac{3745}{5000} \times 100 = 74.9\% \simeq 75\%$ M1W1 13

| | | AVAILABLE MARKS |
|---|---|---|

**11**




(i) $v = u + at$

$\Rightarrow 21 = 0 + 15a$

$\Rightarrow \quad a = \frac{21}{15} = 1.4\ \text{m/s}^2$ MW1

**(ii)** Accelerating force = mass × acceleration

A.F. = (1250 + 550) × 1.4 M1

A.F. = 2520 N W1

Accelerating force = Tractive force – Resistance

$\Rightarrow 2520 = 3840 - R$ M1

$\Rightarrow \quad R = 1320\ \text{N}$ W1

**(iii)** Resistance to motion of the van = 0.704 × 1250

= 880 N MW1

**(iv)** Resistance to motion of the trailer = 1320 – 880

= 440 N MW1

**(v)** Let T = tension in tow-bar

T – 440 = 550 × 1.4 M1

$\Rightarrow$ T = 770 + 440 = 1210 N W1

**(vi)** Retarding force on trailer = 440 N

$\Rightarrow \quad 440 = 550a$ MW1

$\Rightarrow \quad a = \frac{440}{550} = 0.8\ \text{m/s}^2$ W1

Using $v^2 = u^2 + 2as$

$0 = 21^2 - 2(0.8).s$ M1

$\Rightarrow s = \frac{21^2}{1.6} = 275.625 = 276\ \text{m}$ W1 13

| AVAILABLE MARKS |
|---|

**12** **(i)**

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| | 10 | 7 | 1 | 4.5 | 4.5 | 3 | 6 | 8 | 9 | 2 |
| | 10 | 8 | 1 | 3 | 6 | 2 | 4 | 9 | 5 | 7 |
| $d^2$ | 0 | 1 | 0 | 2.25 | 2.25 | 1 | 4 | 1 | 16 | 25 |

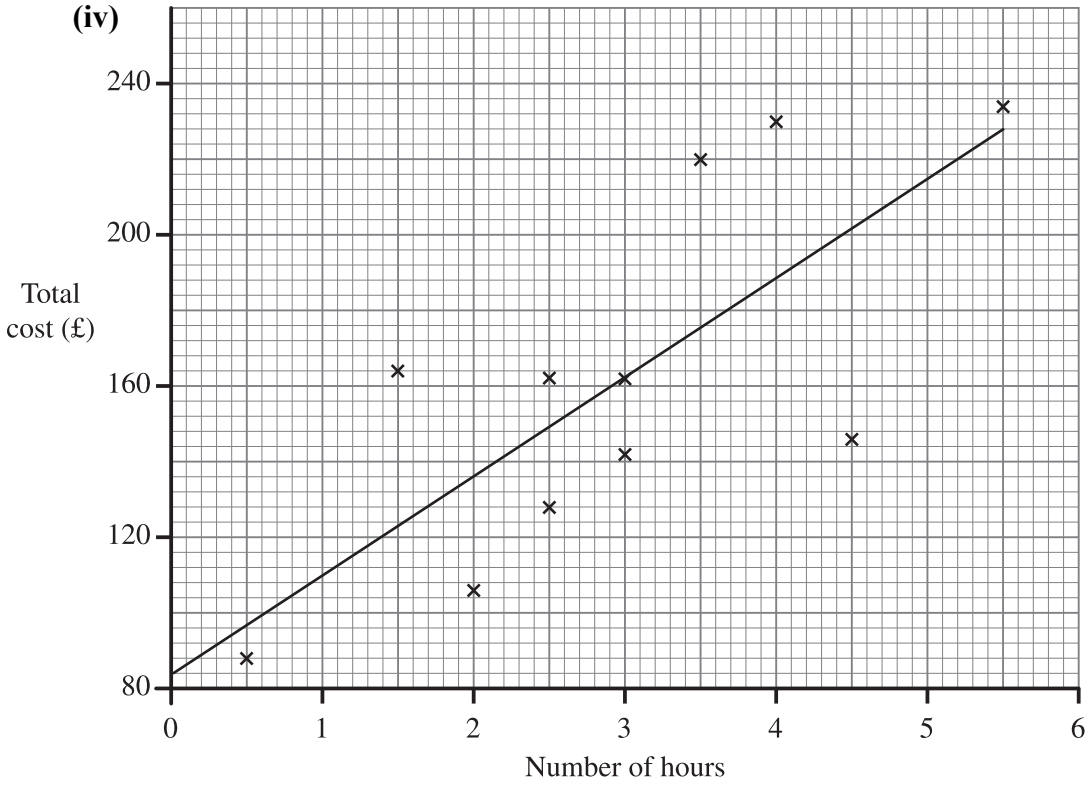
Ranks W1W1

**(ii)** $1 - \dfrac{6(52.5)}{10(99)}$ $\sum d^2$ M1W1

$= 0.68$ M1W1

**(iii)** Positive correlation M1

**(iv)**




mean hours = 2.95 mean cost = £162.30 M1W1

line W1

**(v)** correct method for gradient M1

correct method for intercept M1

$y = 26x + 86$ equation MW1

**(vi)** cost/charge/rate per hour M1 14

**Total** **100**